Status report on string field theory

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Why string field theory ?

In the traditional approach of string theory we have rules how to compute on-shell scattering amplitudes.

Focusing on amplitudes in a given fixed background misses a whole lot of physics. (Instantons, Higgs mechanism etc.)

What is string field theory?

- String field theory is basically a field theory for all the modes of a string at once. As such it can be used for the same purposes as traditional field theory.
- There are many different string field theories: for open, closed, bosonic, supersymmetric strings, covariant or light cone, healthy or sick…

What can SFT be useful for ?

- Good for finding new backgrounds in string theory and for describing dynamical transitions
- Systematic computations of scattering amplitudes
- Study processes involving branes (early universe)
- Compute corrections to superpotential
- Shed light on higher spin fields and their interactions

Plan of the talk:

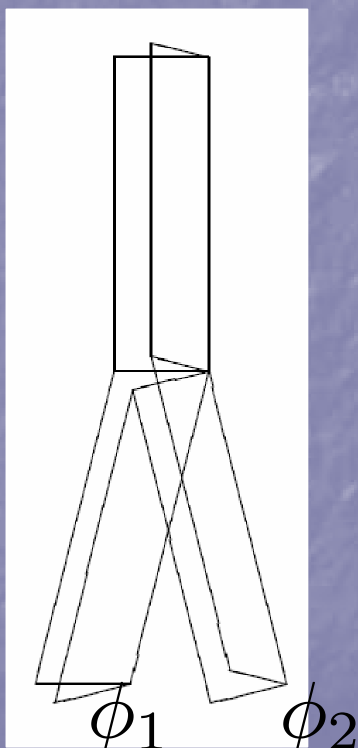
I. On the star product – algebraic treatment

II. Simple exact solutions
 (Tachyon vacuum, marginal deformations)

III. Outlook

Star product is operator product

Let us recall the definition of the star product



The OSFT states look as semi-infinite strips, with operator insertion in the far past

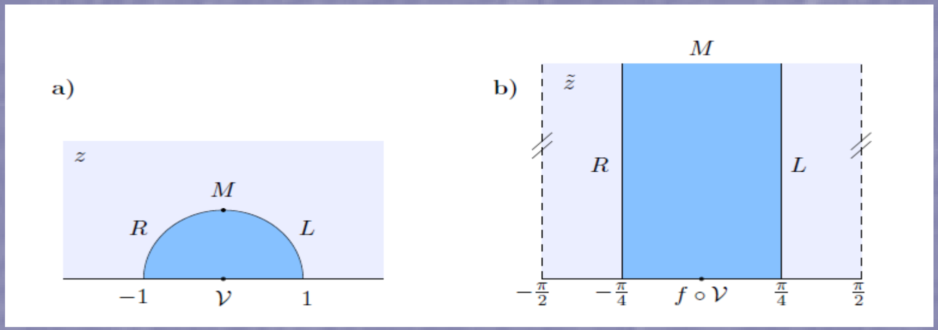
Let us integrate out over the V-shaped part of the worldsheet between ϕ_1 and ϕ_2

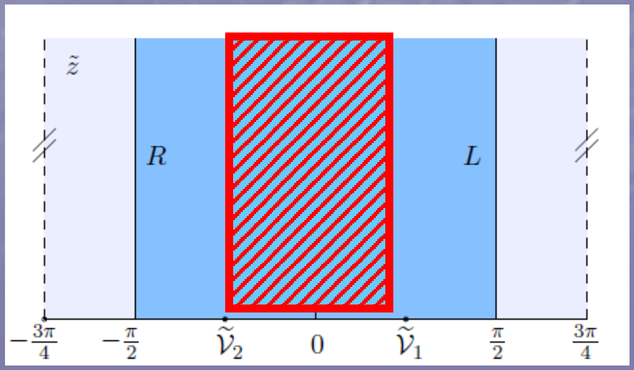
The result can be effectively summarized by the insertion of a (non-local) operator e^{-K}

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle$$

Star product is operator product

To find out what e^{-K} stands for, one can perform a conformal transformation





$$e^{-K} = e^{-\frac{\pi}{2}\int_{-M}^{M} T_{\tilde{z}\tilde{z}} d\tilde{z}}$$

Just like

$$e^{-tL_0} = e^{-t\oint T_{ww} dw}$$

Star product is operator product

Star multiplication is ISOMORPHIC
to operator multiplication

$$|\phi_1\rangle * |\phi_2\rangle = |\phi_1 e^{-K} \phi_2\rangle$$

Introduce new (non-local) CFT operators

$$\hat{\phi} = e^{K/2}\, \phi\, e^{K/2}$$

The associated states satisfy:

$$|\hat{\phi}_1\rangle * |\hat{\phi}_2\rangle = |\widehat{\phi_1 \phi_2}\rangle$$

So $\phi \rightarrow |\hat{\phi}\rangle$ is the isomorphism

Useful truncations of the operator algebras

Trying to restrict the star algebra to some minimal sector, one needs operators

$1,\, e^{-K},\, e^{-2K}, \ldots$

One can also try adding operators

$K, K^2, \ldots$

For tachyon condensation we need the

c tachyon at zero momentum

Useful truncations of the operator algebras

With c and K we can freely generate an algebra subject to the only constraint $c^2 = 0.$

The operators c and K do not commute, their commutator is a new object $[K, c] = \partial c$

The BRST operator Q is not a part of the algebra, but is a derivative operator on the algebra

$$QK = 0, \quad Qc = cKc$$

At this point one can already attempt to solve the equations of motion

$$Q\Psi + \Psi * \Psi = 0$$

It does not take much trying to find the simplest solution is $\Psi = \alpha c - cK$

$$Q\Psi = \alpha(cKc) - (cKc)K$$

$$\Psi * \Psi = \cancel{\alpha^2 c^2} - \cancel{\alpha c^2 K} - \alpha cKc + (cK)(cK)$$

This solution unfortunately does not obey some regularity conditions.

Useful truncations of the operator algebras

To find more general solution we add a new generator B which obeys $QB = K$
It is given by the same expression as K but with energy momentum tensor replaced by the b ghost.
To summarize the relations are:

$$[K, B] = 0, \quad Bc + cB = 1,$$
$$B^2 = 0, \quad c^2 = 0,$$

$$QK = 0, \quad QB = K, \quad Qc = cKc.$$

Useful truncations of the operator algebras

There is one more useful derivative $\mathcal{L}^-$

It acts on the algebra as:

$$\begin{aligned} \mathcal{L}^- c &= -c \\ \mathcal{L}^- K &= K \\ \mathcal{L}^- B &= B \end{aligned}$$

and commutes with Q.

Simple solutions to the e.o.m

Very simple class of solutions is given by

Okawa 2006, M.S. 2005

$$\begin{aligned}\Psi &= Fc\frac{KB}{1-F^2}cF \\ &= (1-FBcF)Q\left(\frac{1}{1-FBcF}\right)\end{aligned}$$

where $F = F(K)$

The solution is a pure gauge unless $\frac{1}{1-FBcF}$ is ill defined.

Simple solutions to the e.o.m

Since

$$\begin{aligned}(1-FBcF)^{-1} &= 1+FBcF+FBcF^2BcF+\cdots \\ &= 1+\frac{F}{1-F^2}BcF\end{aligned}$$

We need

$\frac{1}{1-F^2}$ to be ill defined string field

$\frac{K}{1-F^2}$ to be well defined string field

Homotopy operator

The kinetic operator around the new vacuum Ψ is given by $Q_\Psi = Q + \{\Psi, \bullet\}$

To see whether there is any cohomology or not, around the solution, we may attempt to construct a homotopy operator, such that $\{Q_\Psi, A\} = 1$

Remarkably such an operator formally exists

$$A = \frac{1-F^2}{K} B$$

When is a string field well defined ?

Our criterion is quite conservative and is not meant to be exhaustive. We demand that the state be a non-divergent superposition of geometric states. In terms of F this means that

$$F(K) = \int_0^\infty e^{-Kt} f(t)$$

for some $f(t)$

Simple examples

1) $F(K) = a \quad (const.)$

2) $F(K) = \sqrt{1 - \beta K}$

3) $F(K) = e^{-K}$

4) $F(K) = \frac{1}{\sqrt{1+K}}$

Example I.

For the solution based on $F(K) = a \quad (const.)$
one finds $\Psi = \frac{a}{1-a^2} Q(Bc)$

and the formal homotopy operator is $A = \frac{1-a^2}{K} B$

This is ill defined operator, the energy is also manifestly zero, so this must be the perturbative vacuum !!

Example II.

For the solution based on $F(K) = \sqrt{1 - \beta K}$
one finds (upon a trivial global gauge rotation)

$$\Psi = \beta^{-1} c - cK$$

and the formal homotopy operator is $A = \beta B$

which is well defined.

Therefore, this our first candidate for the tachyon vacuum, unfortunately it is a slightly singular solution.

Example III.

For the solution based on $F(K) = e^{-K/2}$

one finds

$$\Psi = e^{-K/2} cB \left[\frac{K}{1 - e^{-K}} \right] ce^{-K/2}$$

This is the tachyon vacuum found in 2005 (M.S.)

The formal homotopy operator is

$$A = \frac{1 - e^{-K}}{K} B = \int_0^1 dt\, e^{-tK} B$$

which is well defined. The only trouble with the solution is that the inverse Laplace transform of the term in square brackets does not die off fast enough. One finds so called phantom terms.

Example IV.

Finally let us discuss $F(K) = 1/\sqrt{1+K}$

one finds a simple solution Erler, M.S. 2009

$$\Psi = \frac{1}{\sqrt{1+K}} cB(1+K)c \frac{1}{\sqrt{1+K}}$$

and the formal homotopy operator is $A = \frac{B}{1+K}$

which is well defined.

Therefore, this another candidate for the tachyon Vacuum, it is the simplest solution so far.

Example IV.

A simple gauge equivalent version of our solution is

$$\begin{aligned}\Psi &= cB(1+K)c\frac{1}{1+K}\\ &= c\frac{1}{1+K} + Q(Bc)\frac{1}{1+K}\end{aligned}$$

A nice feature of our solution is that it does not have phantom terms and it is easy to compute the energy.

Energy computation

It is very easy:

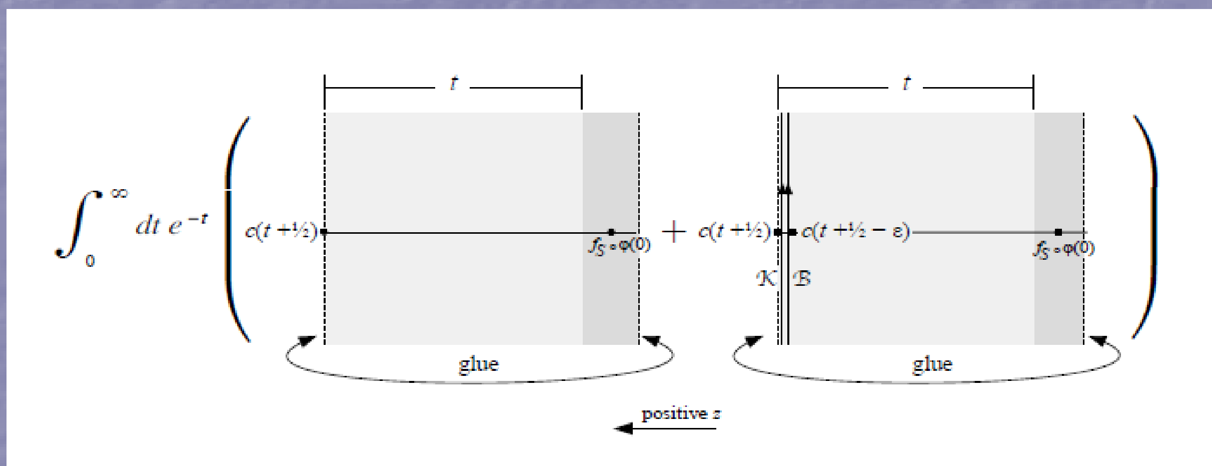
$$-\left(\frac{t_1+t_2}{\pi}\right)^2 \sin^2 \frac{\pi t_1}{t_1+t_2}$$

$$\begin{aligned}
E &= -S = \frac{1}{6}\langle\, \Psi, Q\Psi \,\rangle \\
&= \frac{1}{6}\,\mathrm{Tr}\left[(c + Q(Bc))\frac{1}{1+K}cKc\frac{1}{1+K}\right] \\
&= \frac{1}{6}\int_0^1 dt_1 dt_2 e^{-t_1-t_2}\,\mathrm{Tr}\left[ce^{-t_1K}c\partial ce^{-t_2K}\right] \\
&= -\frac{1}{6\pi^2}\int_0^\infty du u^3 e^{-u}\int_0^1 dv \sin^2\pi v = -\frac{1}{2\pi^2}
\end{aligned}$$

In accordance with Sen's conjecture !

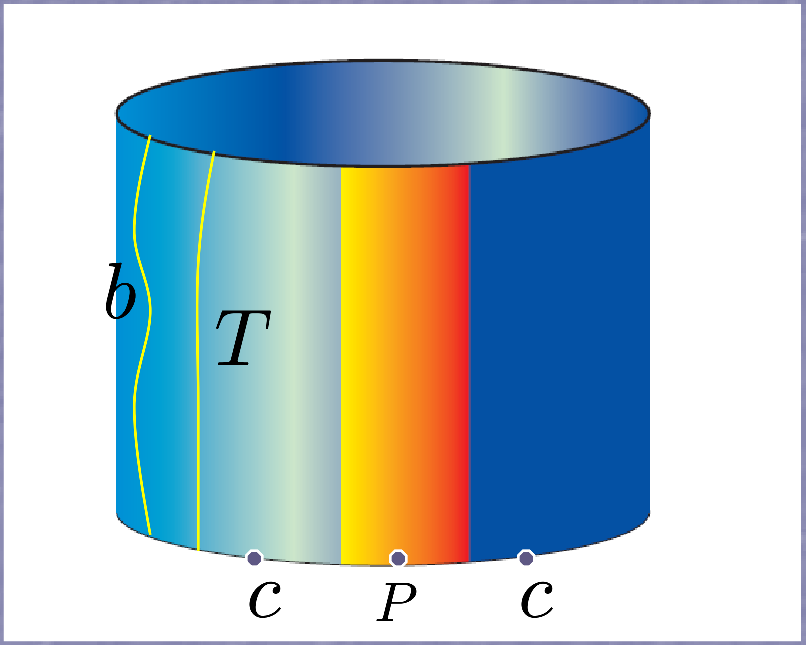
The geometric picture

Our new solution can be presented as



Recall the older solution (M.S. 2005), which is given by a discrete sum over cylinders:

$$\Psi_0 = \sum_{n=0}^{\infty}$$

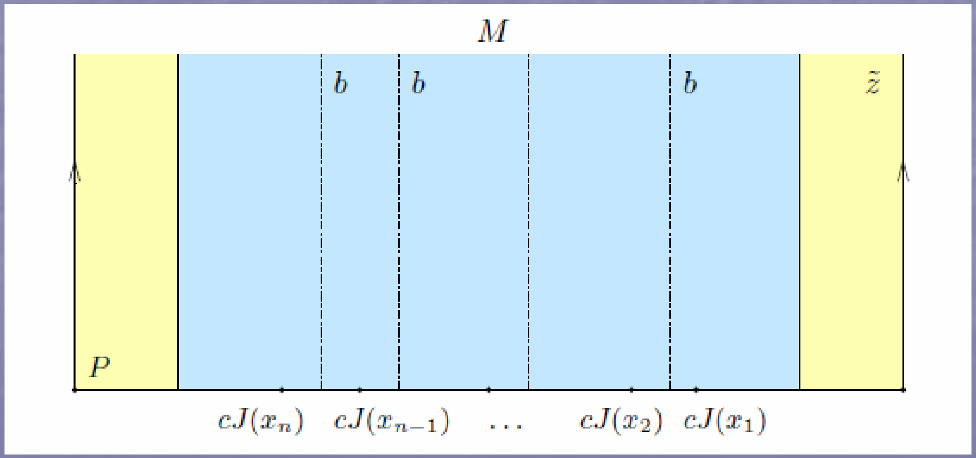


Here the distance of the two c-ghost insertions along the two connecting arcs is π and $\frac{\pi}{2}n$ respectively.

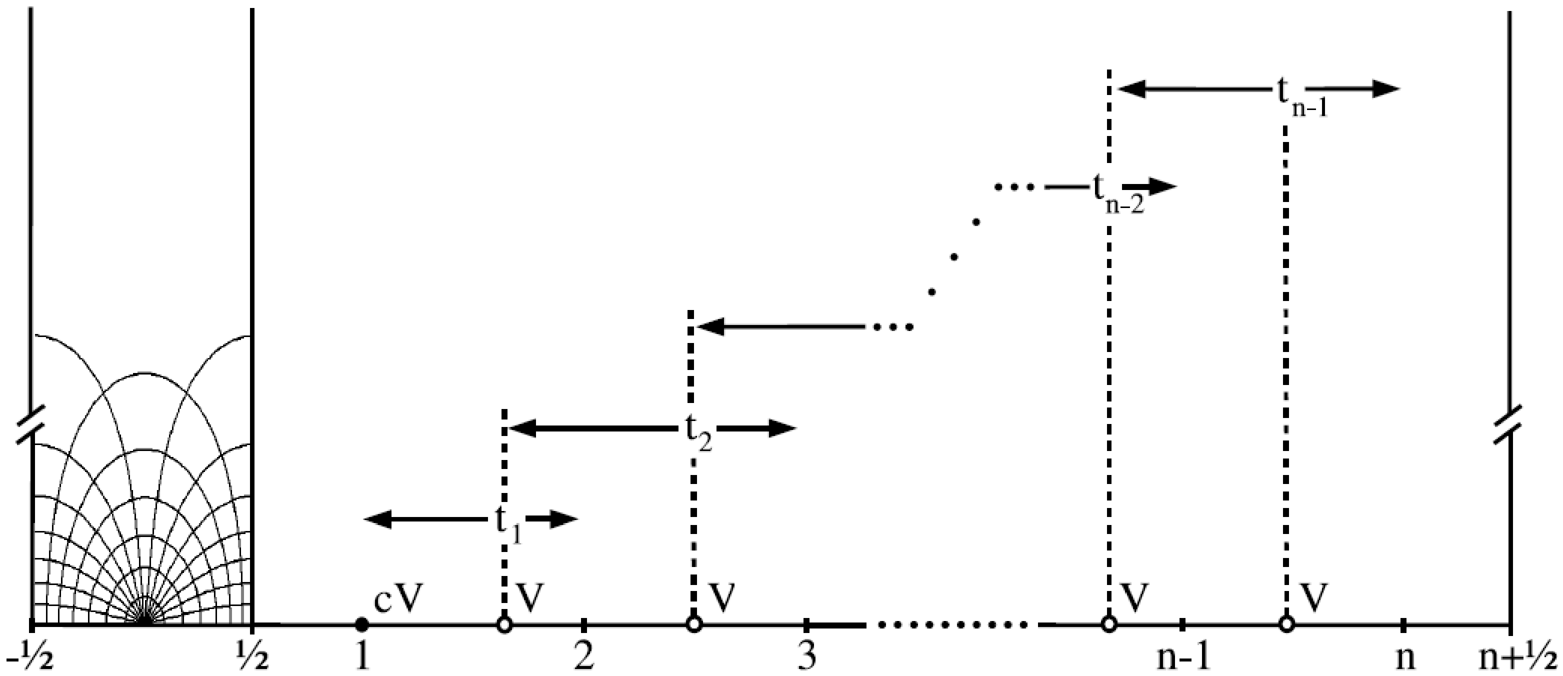
Other solutions

Apart of the tachyon vacuum solution, progress has been made (mostly in 2007) in finding solutions describing marginal deformations:

M.S, Kiermaier, Okawa, Rastelli, Zwiebach, Erler



Kiermaier – Okawa solution



Very close in spirit to CFT, no b-ghosts appear

Concluding remarks

- There are many solutions of OSFT, the tachyon vacuum being the simplest. Many more remain to be found.
- It is very desirable to find direct correspondence between OSFT solutions and boundary states specifying BCFT's. see Zwiebach et al.
- From the string theory point of view, we should generalize our construction to superstrings. Work in progress w/ T. Erler.
- OSFT is one of the finest examples of non-commutative Chern-Simons theory. Can we use some of the math theory to make more progress in string theory? Can we learn about math ?
- OSFT gives a potentially powerful tool to explore the landscape of string theory. It deserves to be studied further.