Quantum Origin of the Universe Structure

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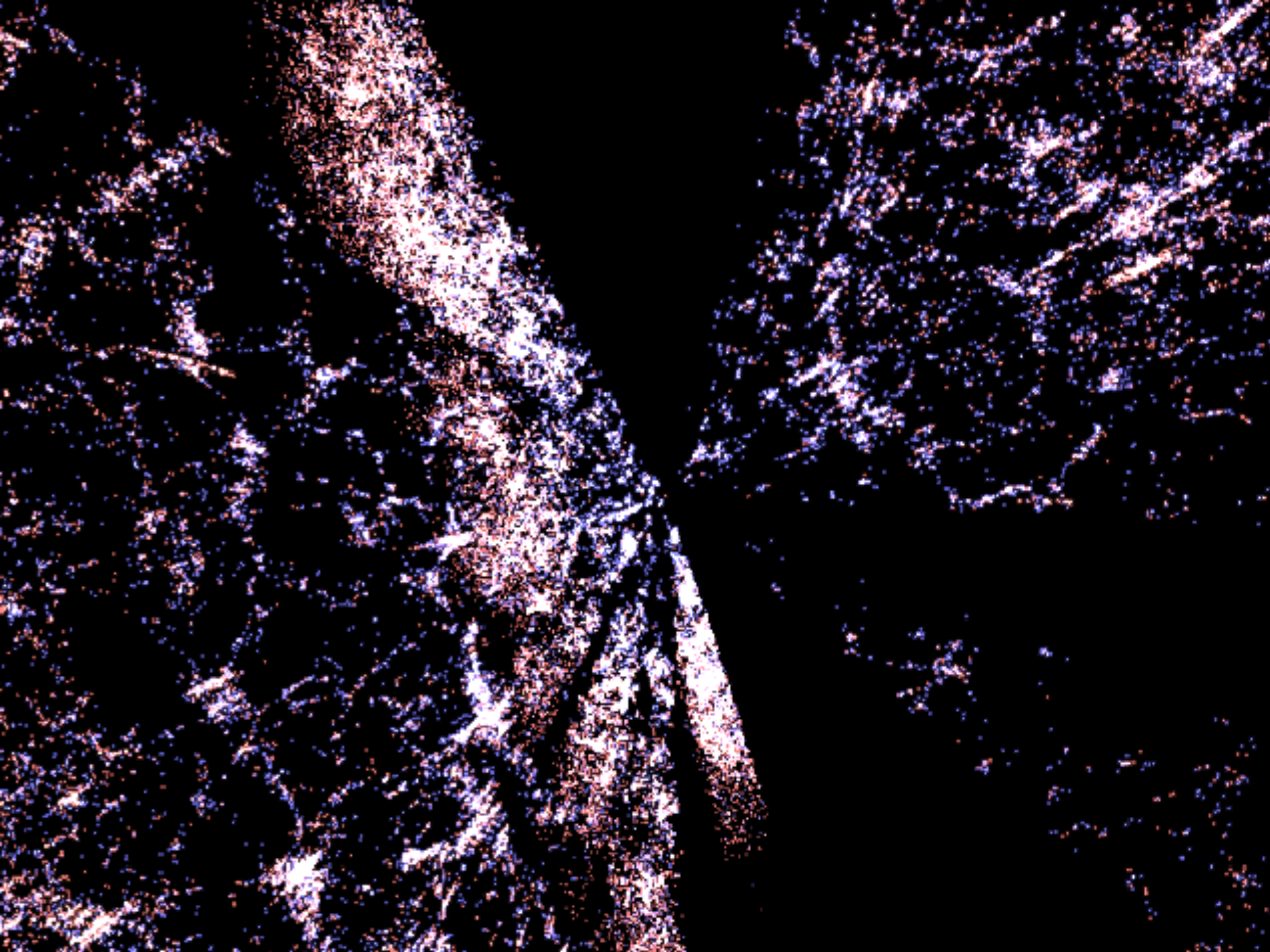
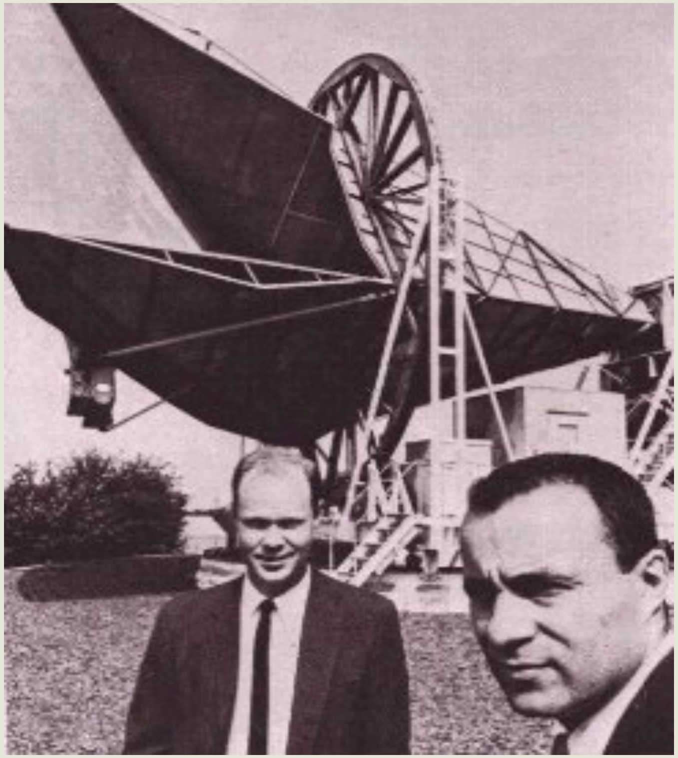
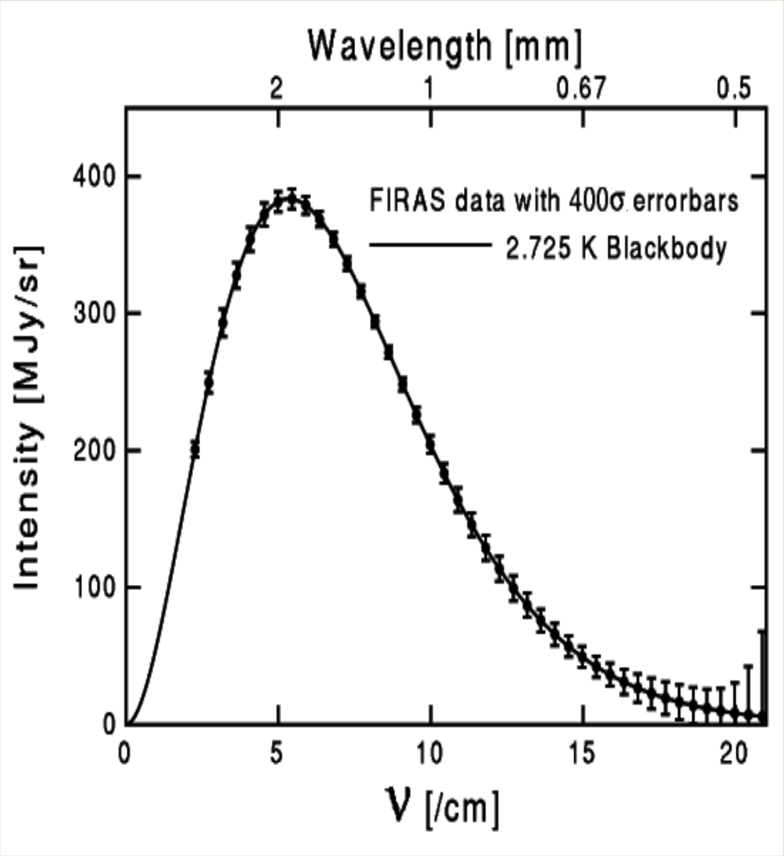


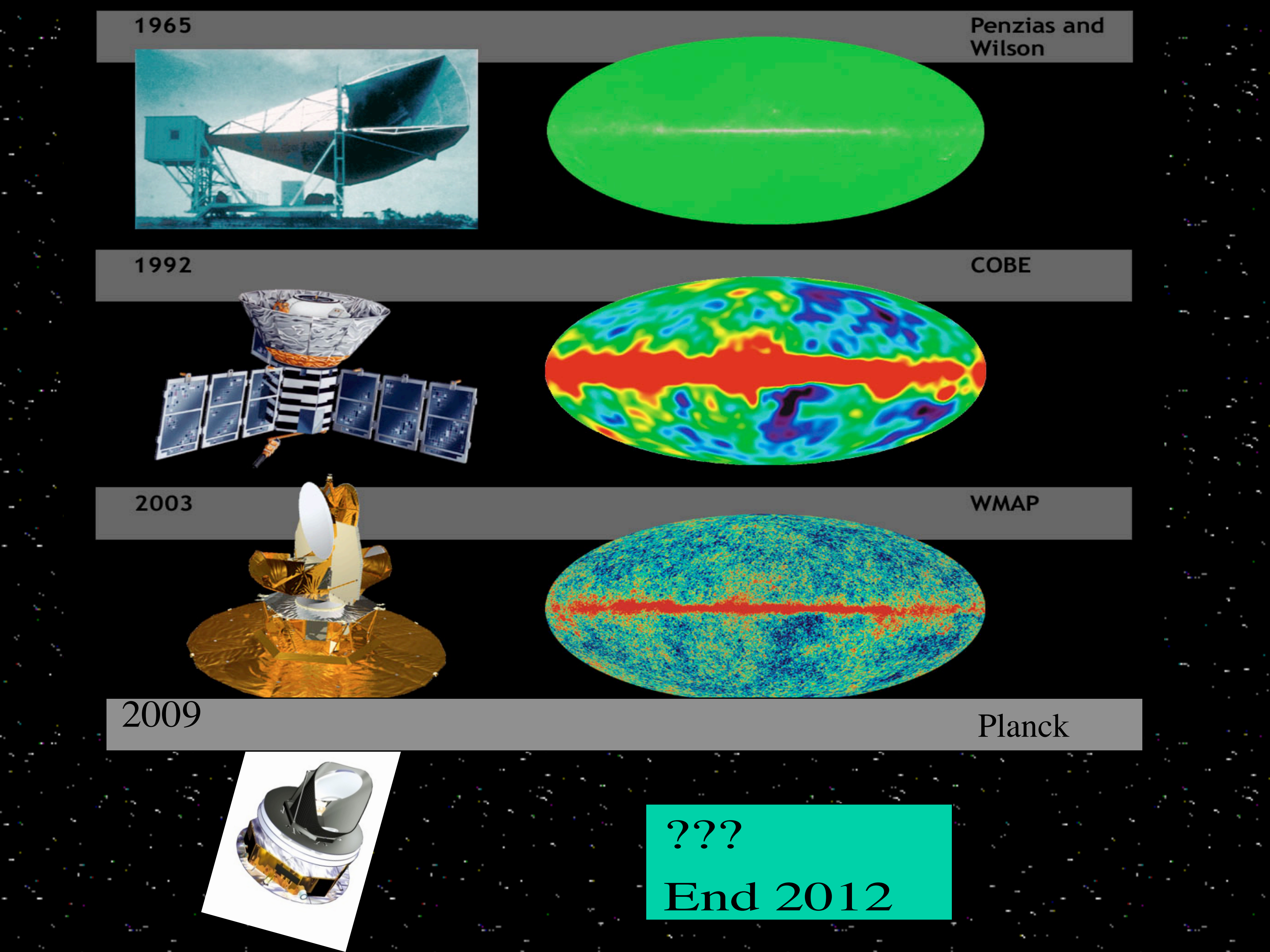
- There exists background radiation with the temperature $T = 2.725° K$



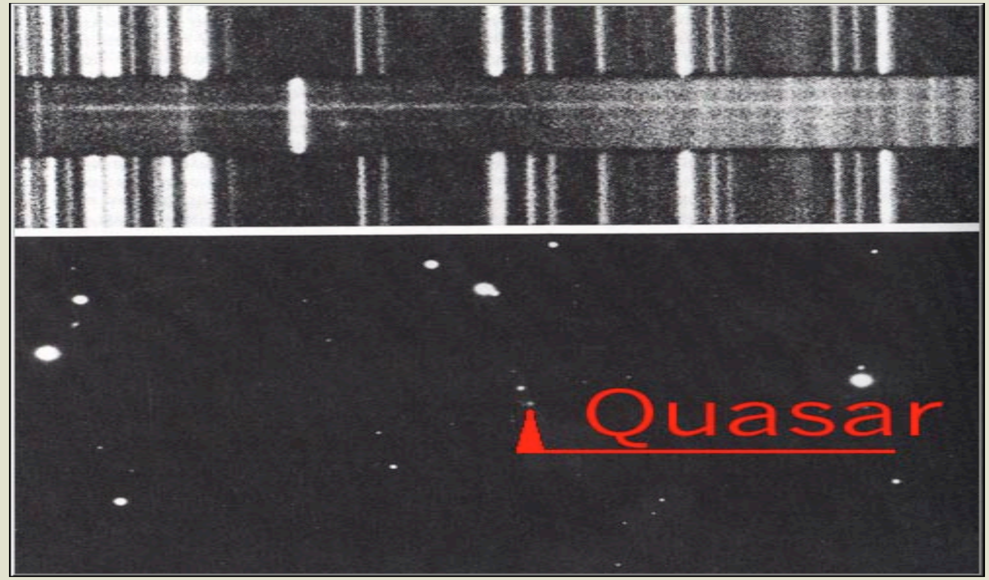
Penzias, Wilson 1965



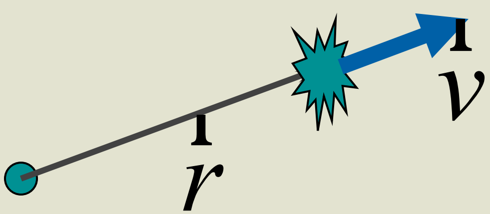
2.725K Blackbody Spectrum of the CMB


1965
Penzias and Wilson
1992
COBE
2003
WMAP
2009
Planck
???
End 2012

The Universe expands



Hubble law



$$\vec{r} = a(t)\,\vec{\chi}_{com}$$

$$\vec{v} = \dot{a}\,\vec{\chi}_{com}$$

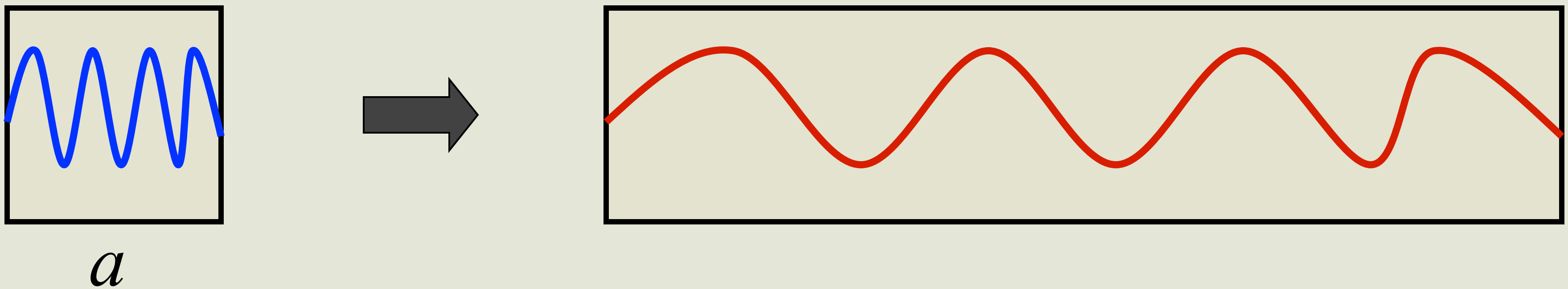
Rate of expansion

$$t \sim \frac{r}{v} \sim 13,7\,bil.\,years$$

Expanding Universe: Facts

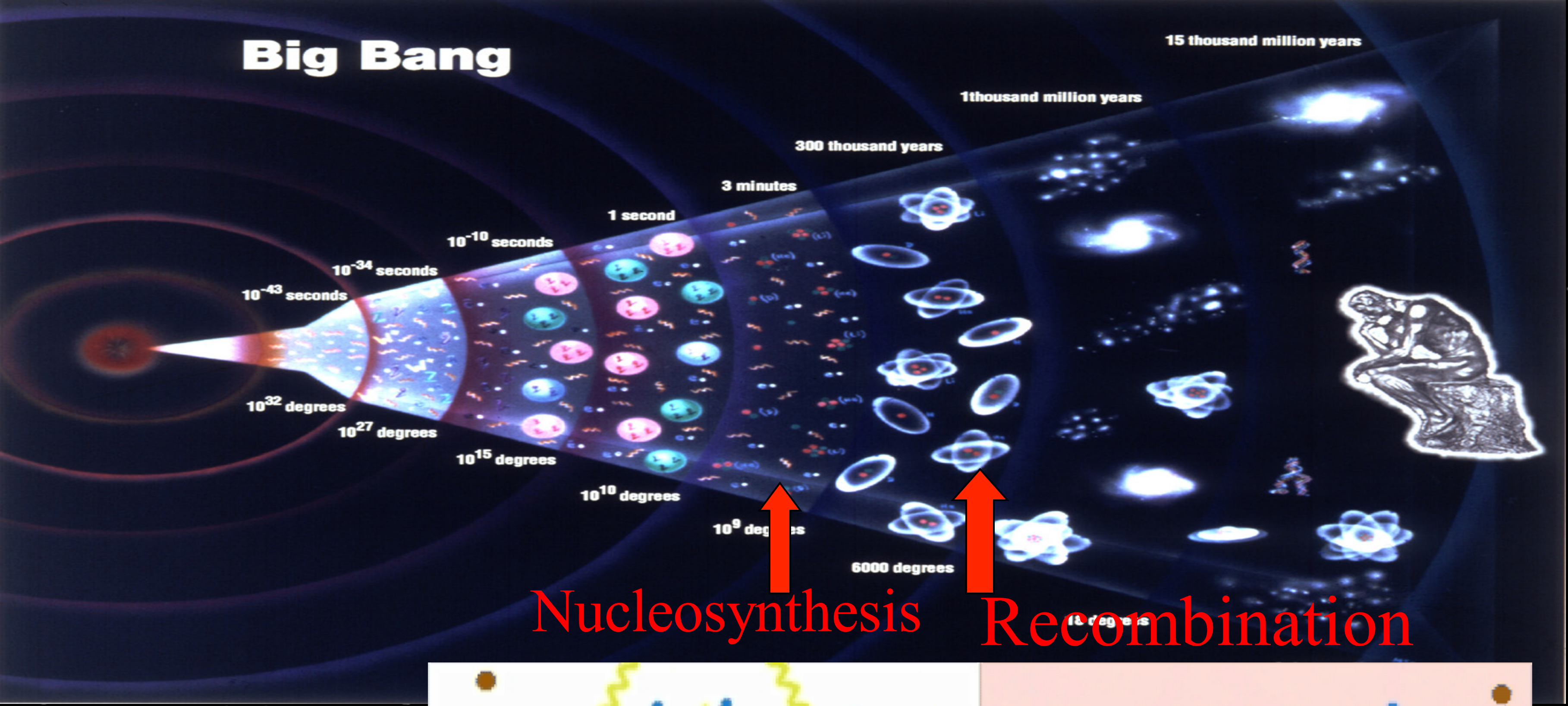
- Today: The Universe is homogeneous and isotropic on scales from 300 millions up to 13 billions light-years
- There exist structure on small scales: Planets, Stars, Galaxies, Clusters of galaxies Superclusters ....
- There is $: \ 75\% H, \ : \ 25\% He$ and heavy elements in very small amounts

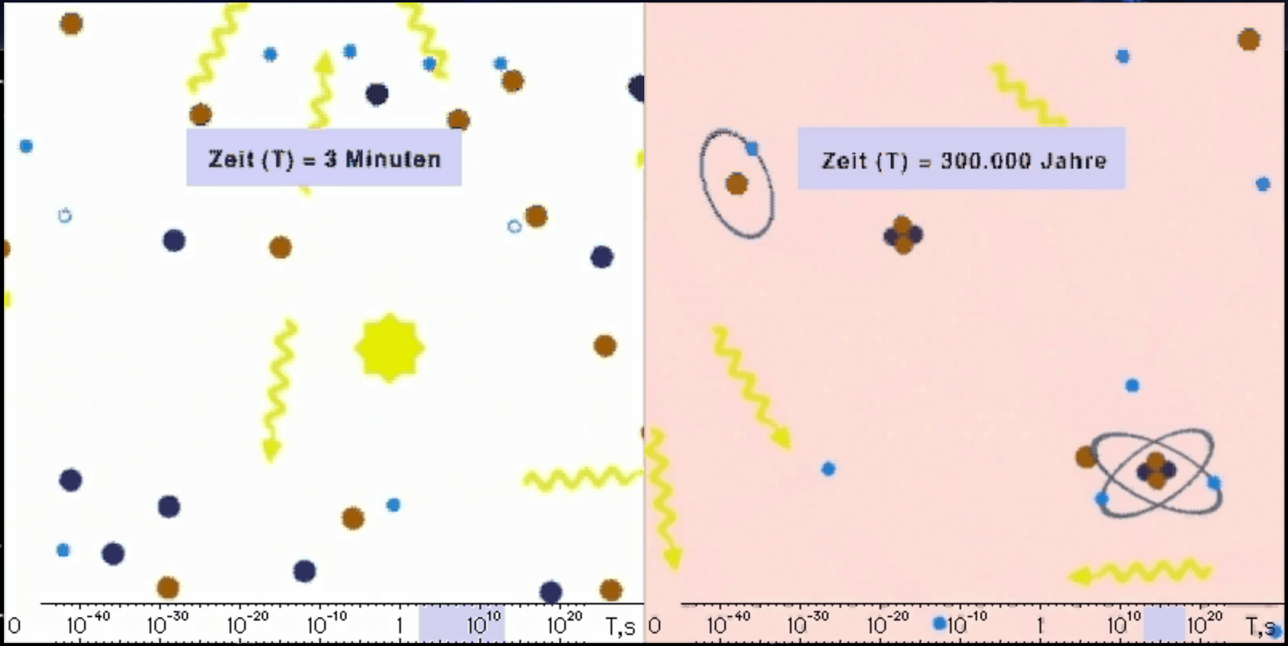
- In past the Universe was VERY hot
- There exist Dark Matter and Dark Energy
- When the Universe was about 1000 times smaller, it was extremely homogeneous and isotropic in all scales $\frac{\delta\varepsilon}{\varepsilon} : 10^{-5}$

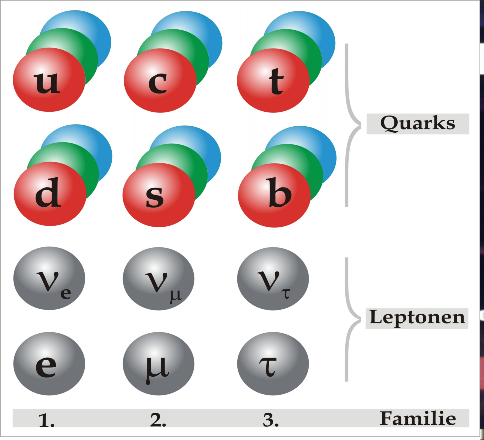


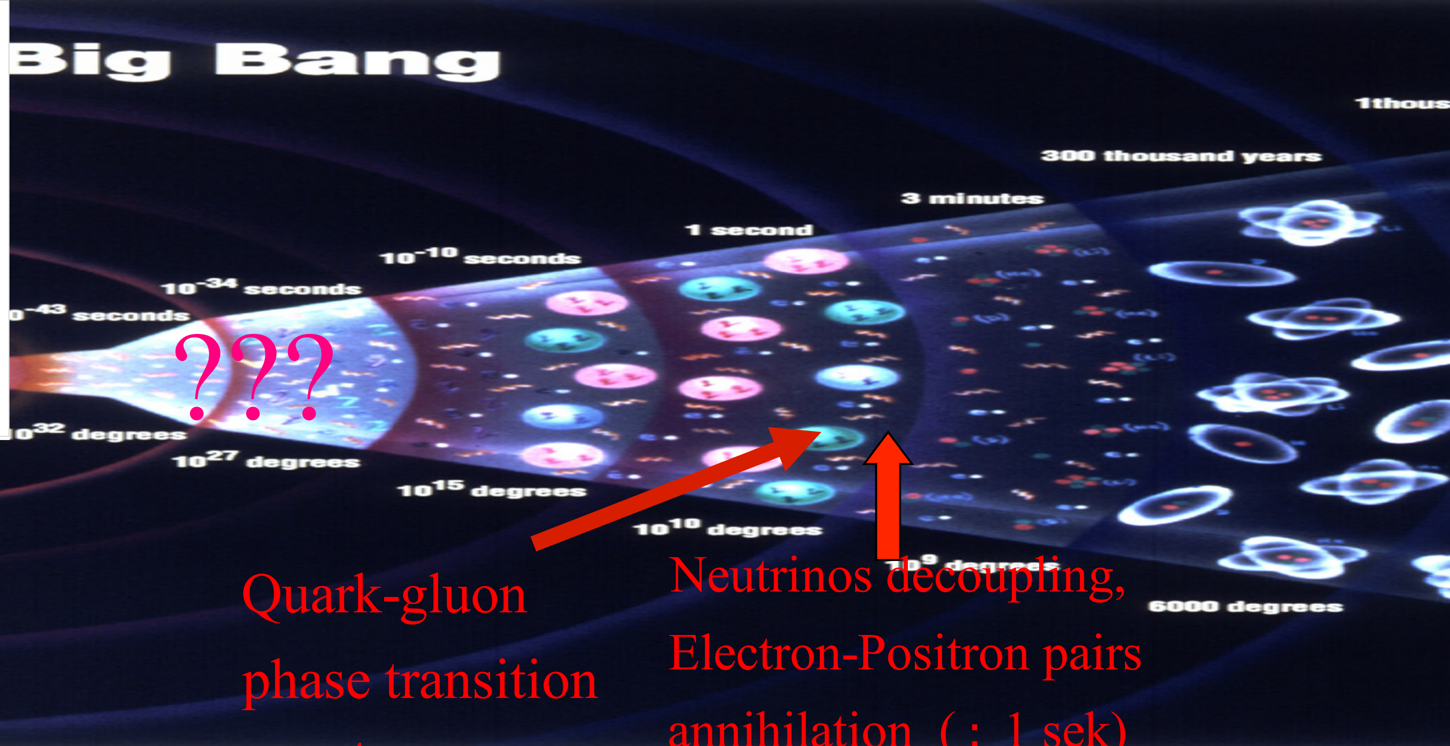
$$\lambda \propto a \quad \Rightarrow \quad T \propto \frac{1}{a}$$

When the Universe was 1000 times smaller its temperature was about $2725°K$


Big Bang
15 thousand million years
1thousand million years
300 thousand years
3 minutes
1 second
10-10 seconds
10-34 seconds
10-43 seconds
10^32 degrees
10^27 degrees
10^15 degrees
10^10 degrees
6000 degrees
Nucleosynthesis
Recombination


Zeit (T) = 3 Minuten
Zeit (T) = 300.000 Jahre
0 10-40 10-30 10-20 10-10 1 10^10 10^20 T,s
0 10-40 10-30 10-20 10-10 1 10^10 10^20 T,s



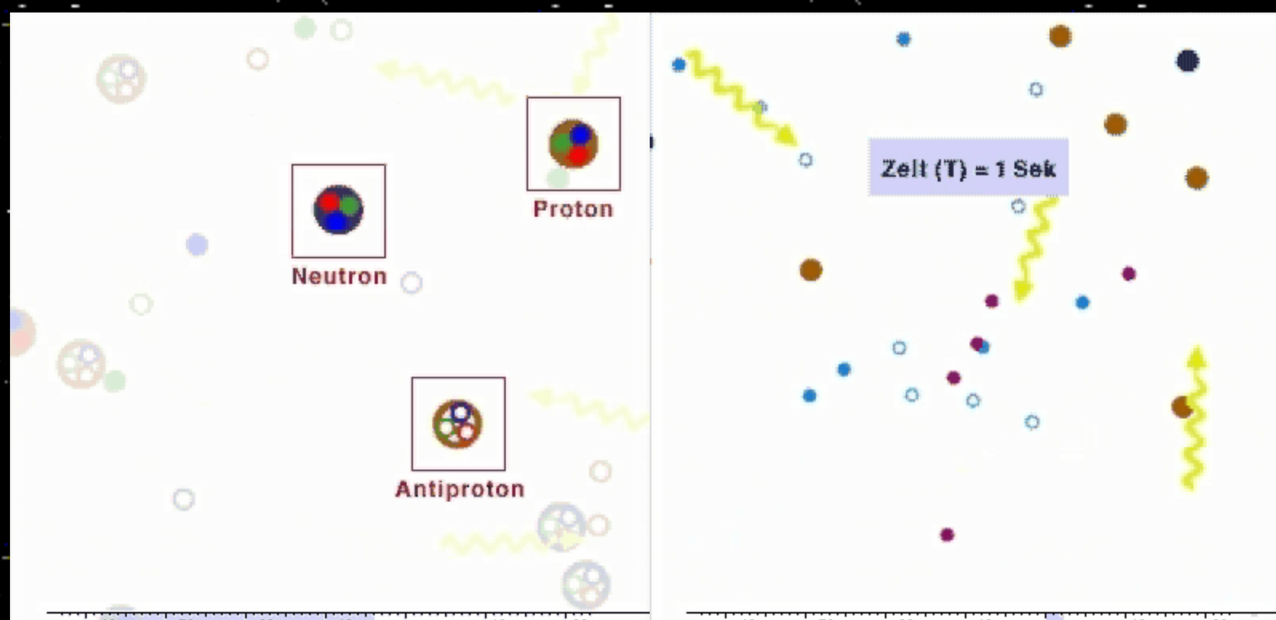


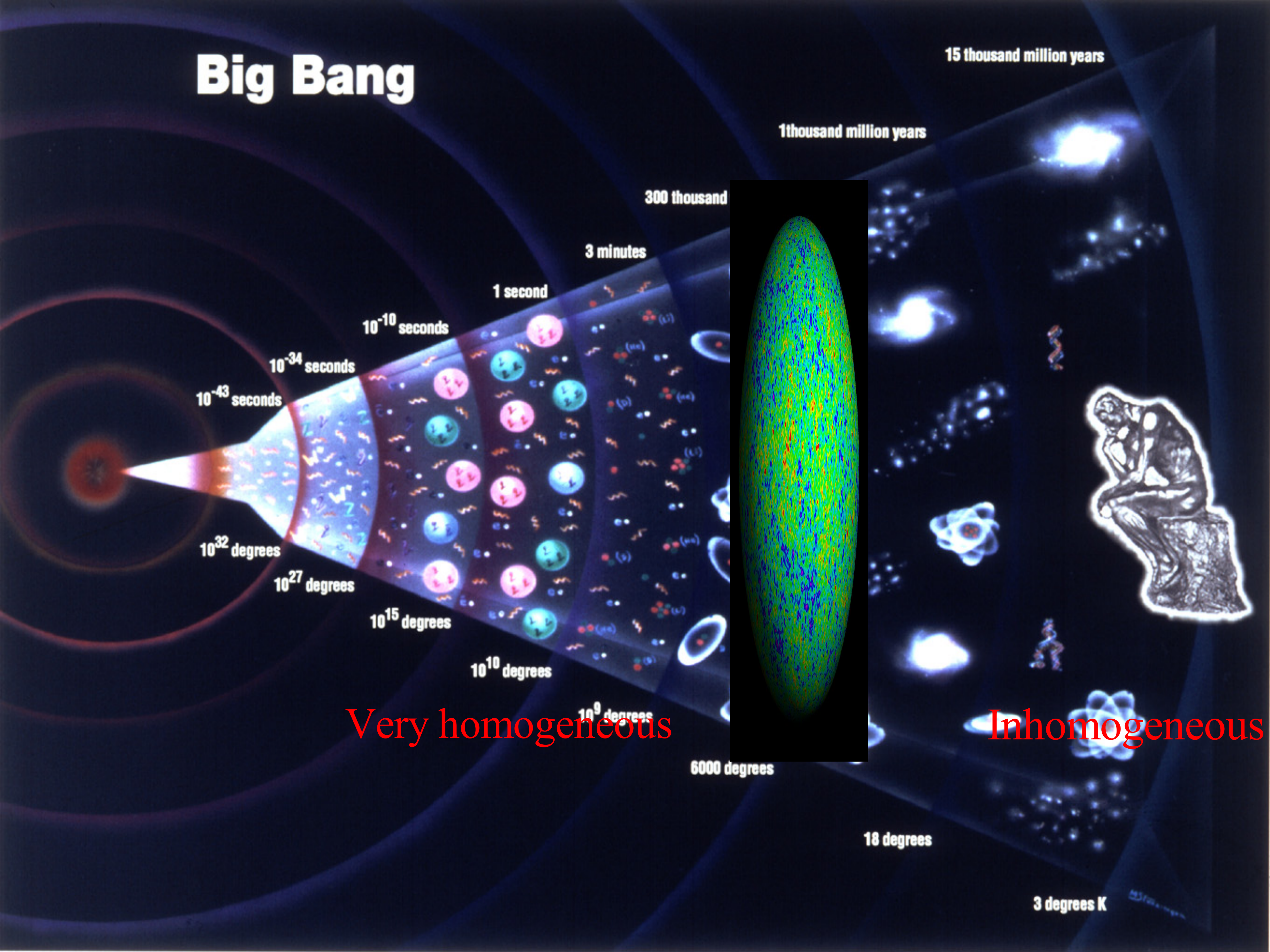
Quark-gluon phase transition : 10^{-4} sec

Neutrinos decoupling, Electron-Positron pairs annihilation (: 1 sek)

t : 10^{-10}

Electro-weak phase transition



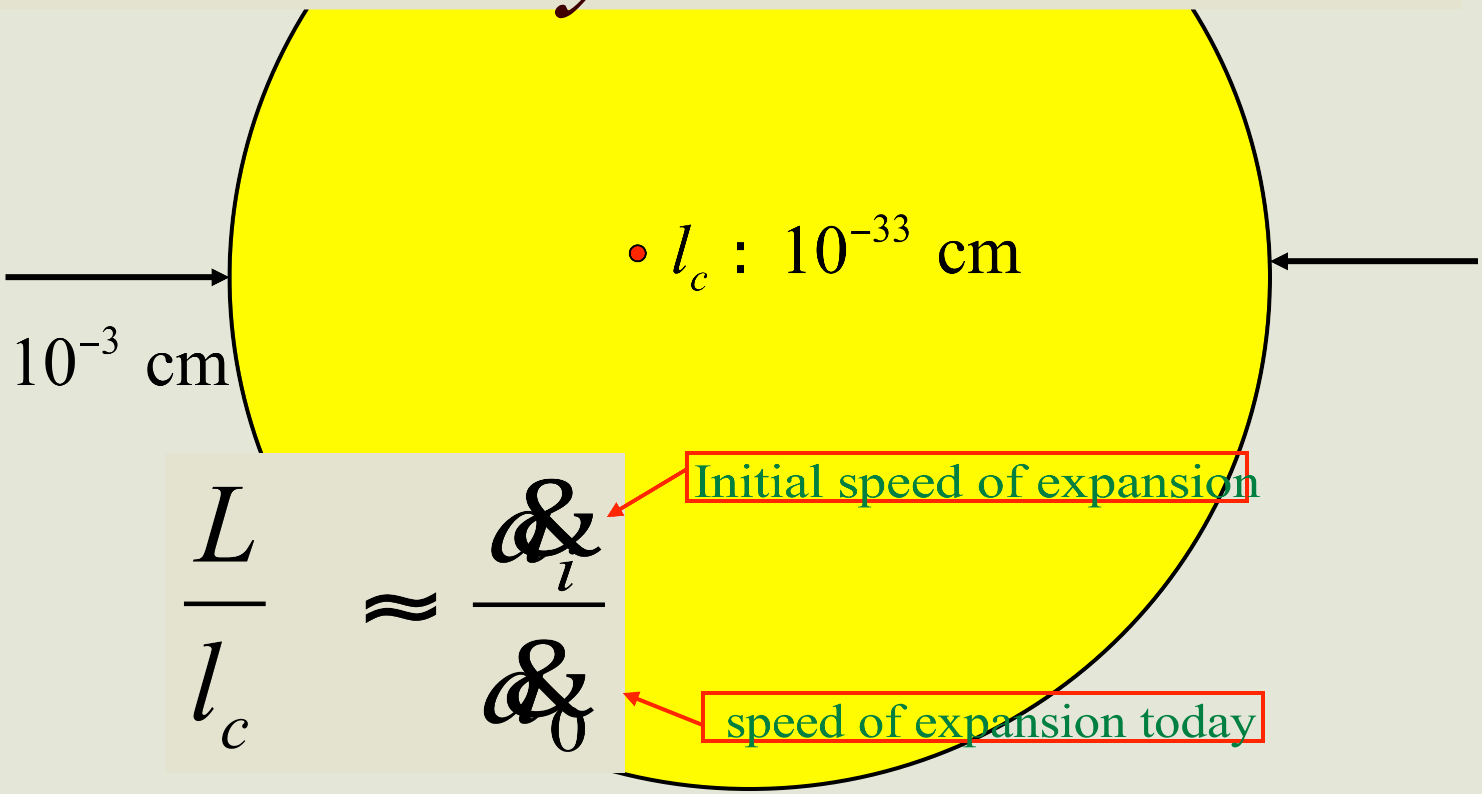


Big Bang
15 thousand million years
1thousand million years
300 thousand
3 minutes
1 second
10^{-10} seconds
10^{-34} seconds
10^{-43} seconds
10^{32} degrees
10^{27} degrees
10^{15} degrees
10^{10} degrees
10^{9} degrees
6000 degrees
18 degrees
3 degrees K
Very homogeneous
Inhomogeneous

INFLATION

In 10^{90} causally disconnected regions $\delta\varepsilon / \varepsilon \leq 10^{-5}$!!!

Why?



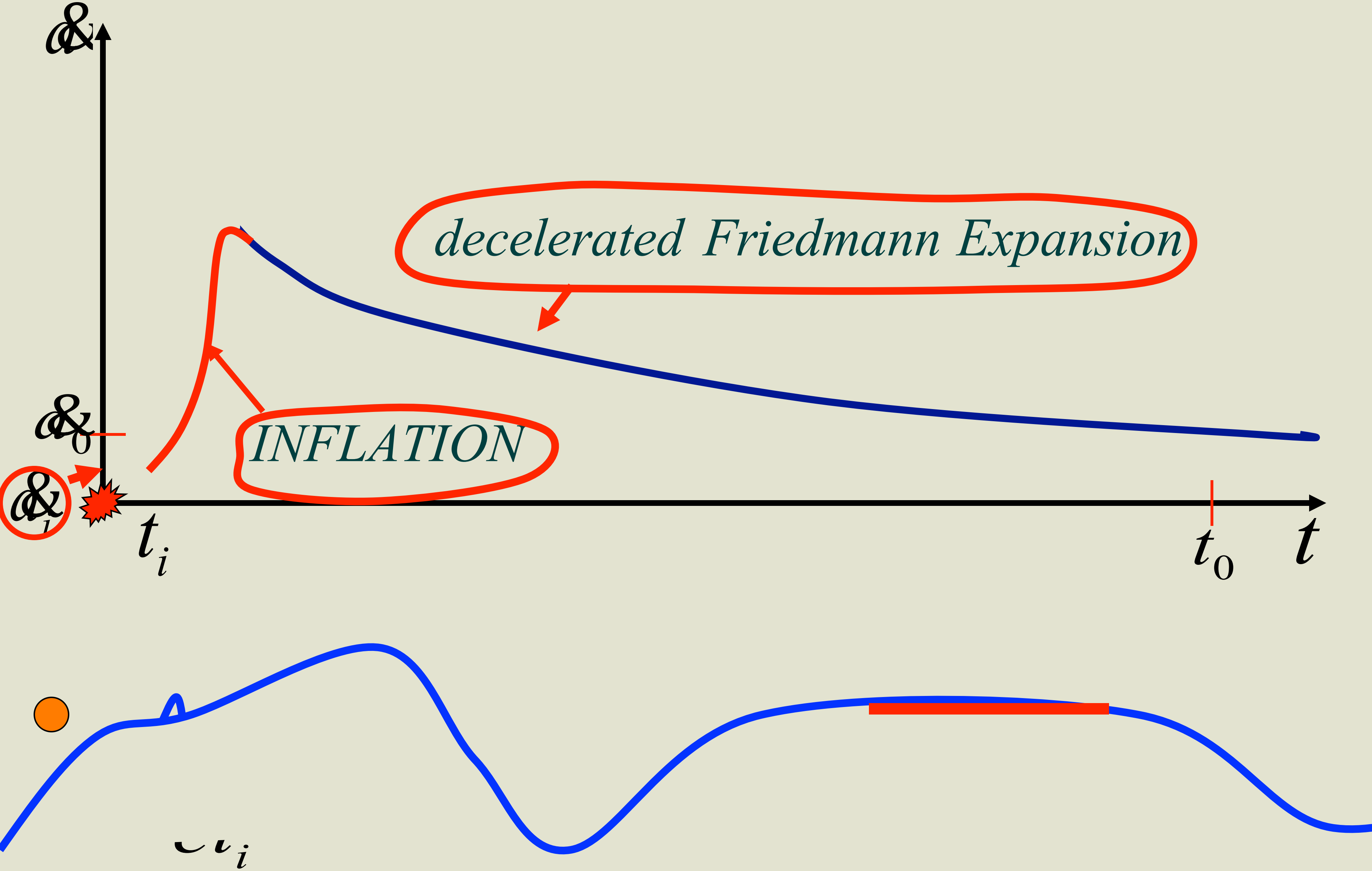
$$\frac{L}{l_c} \approx \frac{\dot{a}_i}{\dot{a}_0}$$

- Reason: Gravitation is attractive force $\Rightarrow \dfrac{\dot{a}_i}{\dot{a}_0} << 1$

- Assumption: Gravity was REPULSIVE during during some time interval in very early Universe

$\Rightarrow$ $\dot{a}_i / \dot{a}_0 \leq 1 \Rightarrow$ *NO* Problem

INFLATION is the stage of accelerated expansion in the very early Universe


decelerated Friedmann Expansion
INFLATION
t_i
t_0
t

- $$\Omega_0 \equiv \frac{\left| E_0^{pot} \right|}{E_0^{kin}} = \frac{1000000009}{1000000003} = 1$$

$$E^{kin} = 3 + 1000000000$$

$$E^{pot} = -9 - 1000000000$$

Prediction

of inflation !

- How gravity can become "repulsive"?

$$\ddot{a} = -\frac{4\pi G}{3}\left(\varepsilon + 3p\right)a$$

acceleration — energy density — pressure

Only if $\varepsilon + 3p < 0 \Rightarrow \ddot{a} > 0$ ≡ "antigravity"

Scenarios

???

Energy density ε, pressure p

$p(\varepsilon)$ – equation of state

$p + \varepsilon = \varepsilon$ for inflation

$$p \approx -\varepsilon$$

Which concrete scenario was realized ???

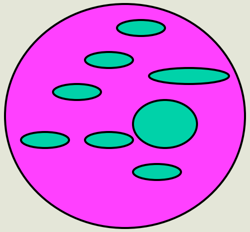
Quantum Fluctuations and Galaxies

- Inflation "wash out" all existing classical inhomogeneities and leaves classical desert

Where the ihhomogeneities come from?

$$\Delta p \Delta x \geq h$$

There always exist unavoidable Quantum Fluctuations



Quantum fluctuations in the density distribution are large (10^{-5})

only in extremely small scales ($\sim 10^{-33}$ cm),

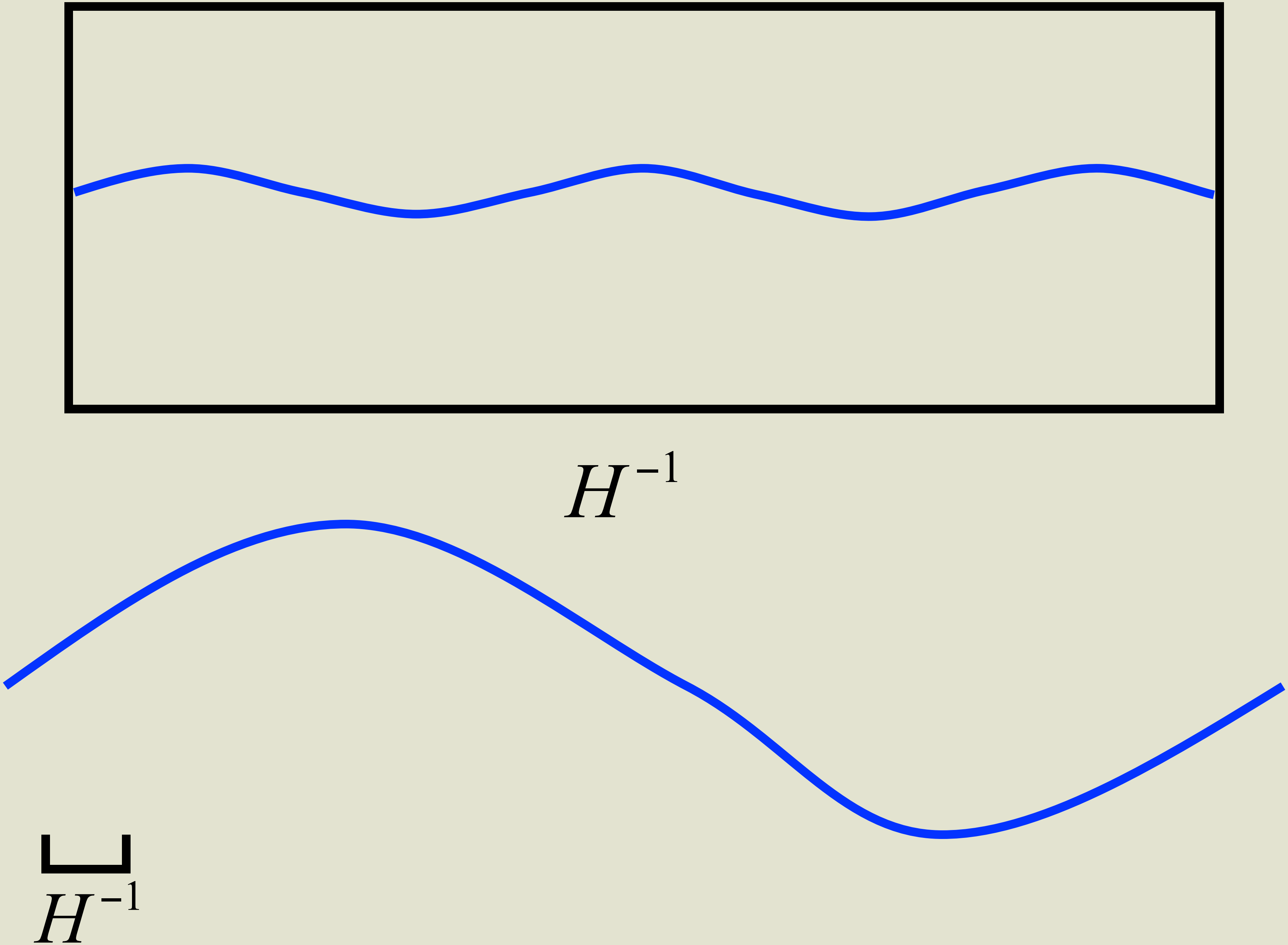
but very small ($\sim 10^{-58}$) on galactic scales ($\sim 10^{25}$ cm)

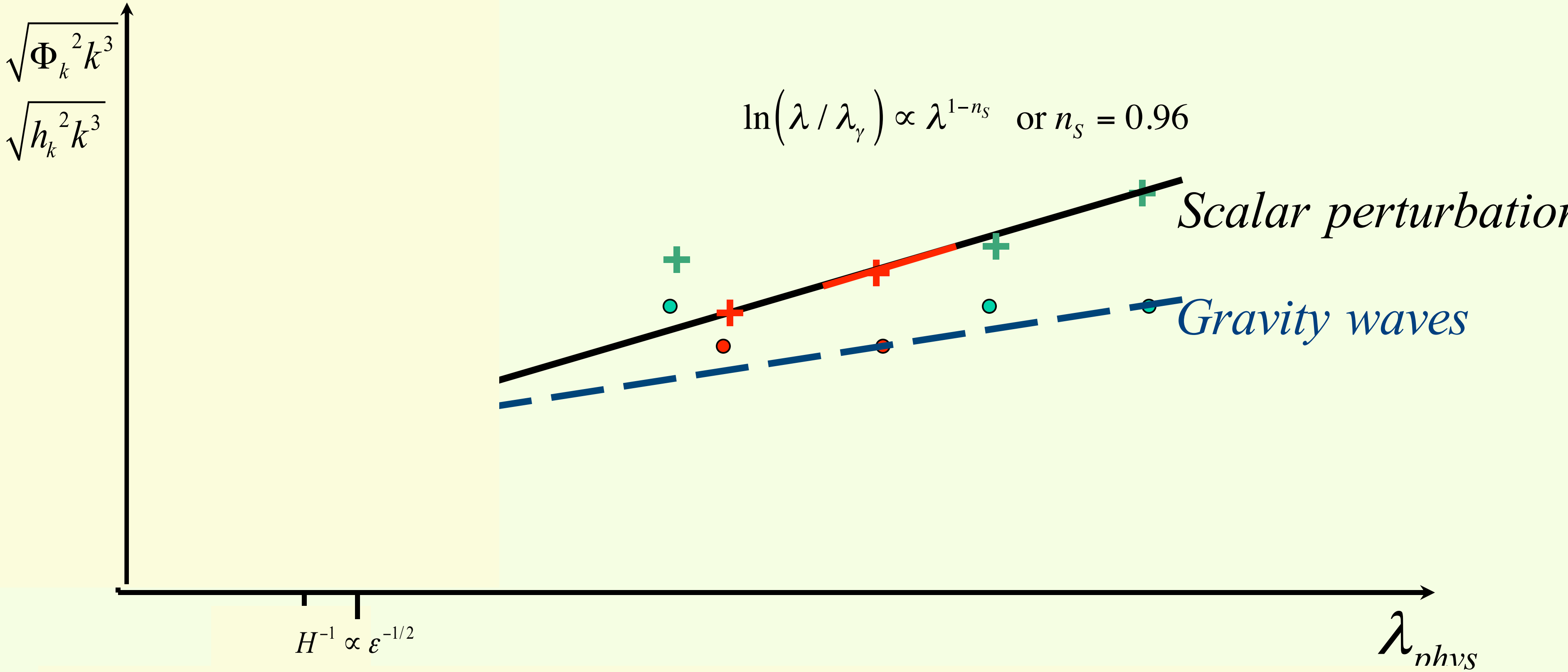
Can we transfer the large fluctuations from extremely small scales to large scales???

Yes !!! but only if in past the Universe went through the stage of accelerated expansion (inflation)

- Consider plane wave perturbation: $\delta\varphi, \Phi \propto \exp\left(i\vec{k}_{com}\vec{x}\right)$

For given k_{com}, $\lambda_{ph}(cm) \propto a / k_{com} \propto a(t)$ and the change of the amplitude with time depends on how big is λ_{phys} compared to the curvature scale (size of Einstein lift) $H^{-1} = a/\dot{a}$


H −1
H −1



$$0.95 < n_S < 0.97$$

$$\frac{T}{S} = O(1)\left(1 + \frac{p}{\varepsilon}\right)^{1/2}_{k \approx Ha}$$

JETP Lett, Vol. 33, No.10, 20 May 1981

Quantum fluctuations and a nonsingular Universe

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P. N. Lebedev Physics Institute, Academy of sciences of the USSR

(Submitted 26 February 1981; 15 April 1981)

Pis'ma Zh. Eksp. Theor. Fiz. 33, No.10, 549-553 (20 May 1981)

Adopting a perturbation of the curvature scalar as a physical variable, we find the corresponding action in the form[6]

$$\delta S_b = \frac{1}{2}\int d^4x \left[\phi'^2 - \nabla^\alpha\phi\nabla_\alpha\phi + \left(\frac{a''}{a} + M^2 a^2\right)\phi^2\right], \quad (5)$$

where $\phi = 1/\sqrt{18\,(4H^2 - M^2)}\; a\delta R/M\ell$, and $\ell = (8\pi G/3)^{1/2} = 4.37\times 10^{-33}$ cm is the Planck length.

A finite duration of the de Sitter stage does not by itself rule out the possibility that this stage may exist as an intermediate stage in the evolution of the universe. An interesting question arises here: Might not perturbations of the metric , which would be sufficient for the formation of galaxies and galactic clusters, arise in this stage? To answer this question, we need to calculate the correlation function for the fluctuations of the metric after the universe goes from the de Sitter stage to the hydrodynamic stage. By analogy with (6) we find

$$\left\langle 0\left|\hat{h}\,(\mathbf{x})\,\hat{h}\,(\mathbf{x}+\mathbf{r})\right|0\right\rangle = \frac{1}{2\pi^2}\int Q^2\,(k)\,\frac{\sin kr}{kr}\frac{dk}{k}, \quad (8)$$

where $h = h^\alpha_\alpha$ and where, for the most interesting region, $H > k > H\exp\left(-3H^2/M^2\right)\left(M^2 \ll H^2\right)$,

$$Q\,(k) \approx 3\ell M\left(1 + \frac{1}{2}\ln\frac{H}{k}\right). \quad (9)$$

The fluctuation spectrum is thus nearly flat. The quantity $Q\,(k)$ is the measure of the amplitude of perturbations with scale dimensions $1/k$ at the time the universe begins the ordinary Friedmann expansion. With $\ell M \sim 10^{-3} - 10^{-5}$ and $M/H \leq 0.1$–these values are consistent with modern theories of elementary particles-the amplitude of the perturbations of the metric on the

From GR content ⇒ one postulate ≡ stage of accelerated expansion ⇒

explanation of hom, isotr.

+ 2 nontrivial predictions

- $\Omega_{total} = 1 \pm 10^{-5}$

- spectrum of pertur.

spectrum is <u>never</u> HZ for generic inflation. It is tilded

$$\Phi_\lambda^2 \simeq \frac{\varepsilon}{\varepsilon_{pl}} \frac{1}{\underbrace{1+p/\varepsilon}_{1/50}}\Bigg|_{\lambda^{-1}\simeq Ha} \qquad (\varepsilon \leftarrow 10^{-12})$$

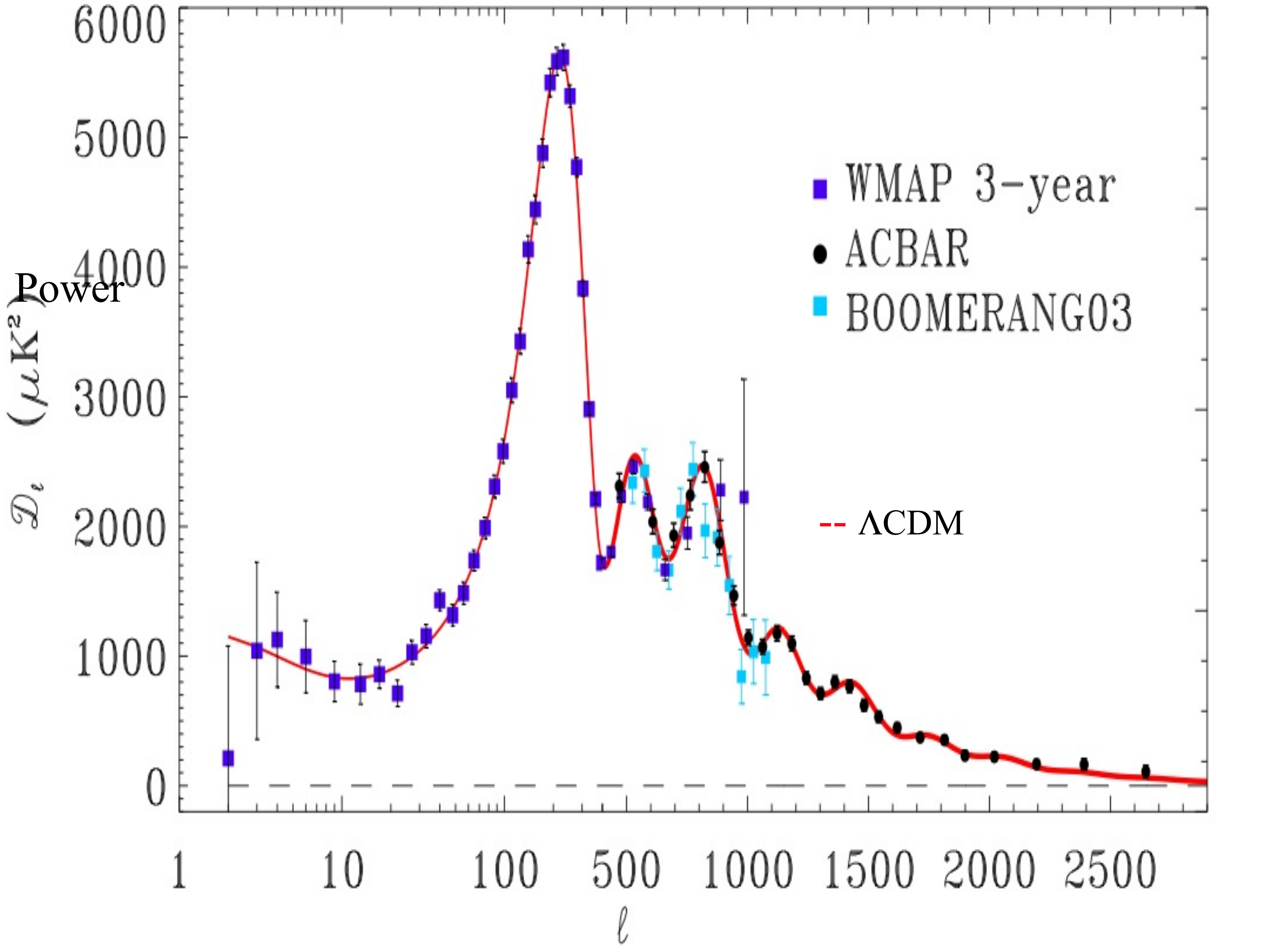
$$n_s - 1 = -3\left(1+\frac{p}{\varepsilon}\right)_{\lambda^{-1}\simeq Ha} - \frac{1}{H}\left(\ln\left(1+\frac{p}{\varepsilon}\right)\right)^{\cdot}$$

(each term < 0)

$$0.9 \overset{?}{\lesssim} n_s < 0.96 \quad !$$

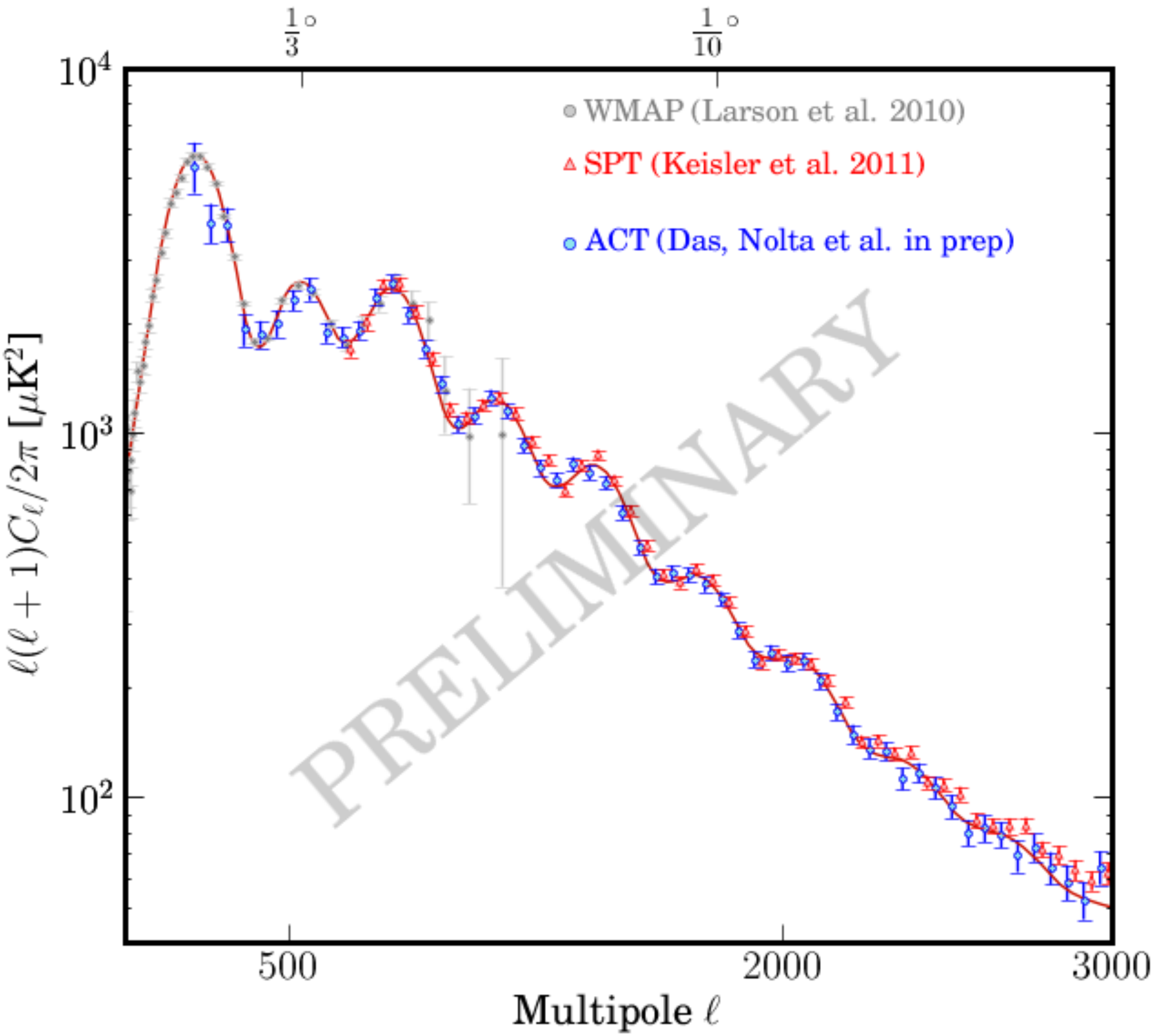
not too much grav. waves!

Gaussian perturbations



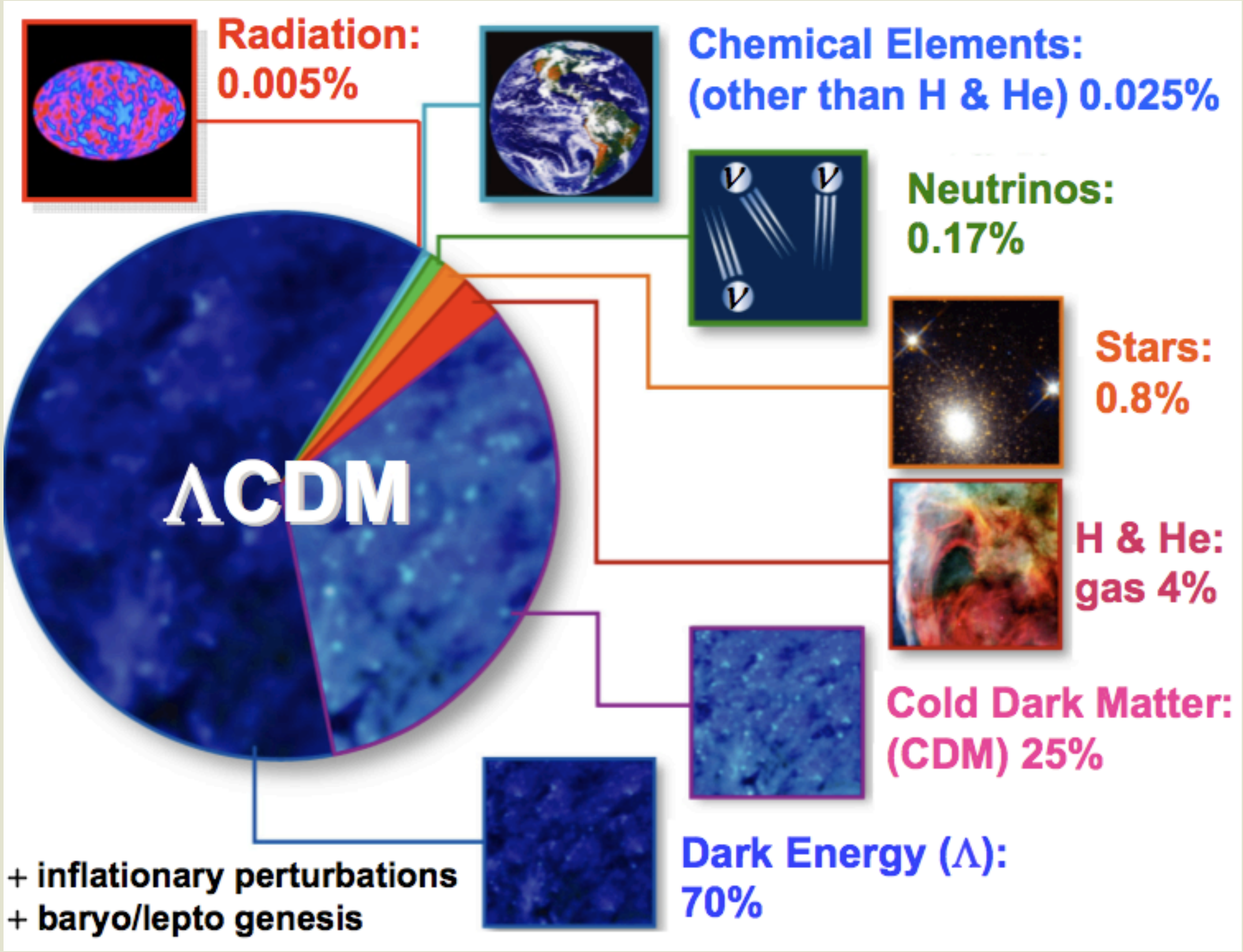
6000
5000
4000
3000
2000
1000
0
Power
$\mathcal{D}_\ell$ (μK^2)
WMAP 3-year
ACBAR
BOOMERANG03
-- ΛCDM
1
10
100
500
1000
1500
2000
2500
ℓ

Small-scale spectrum: summer 2011



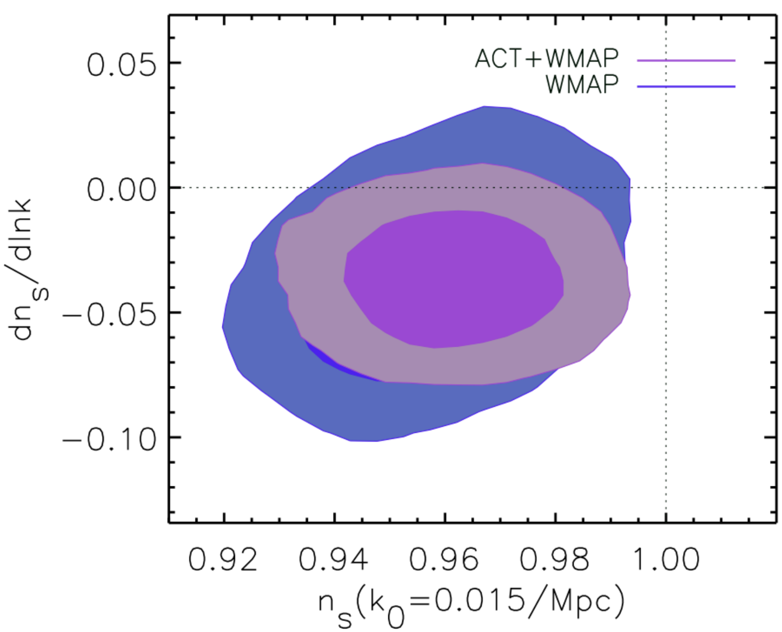
$$\Omega = 1 \pm 0.02$$

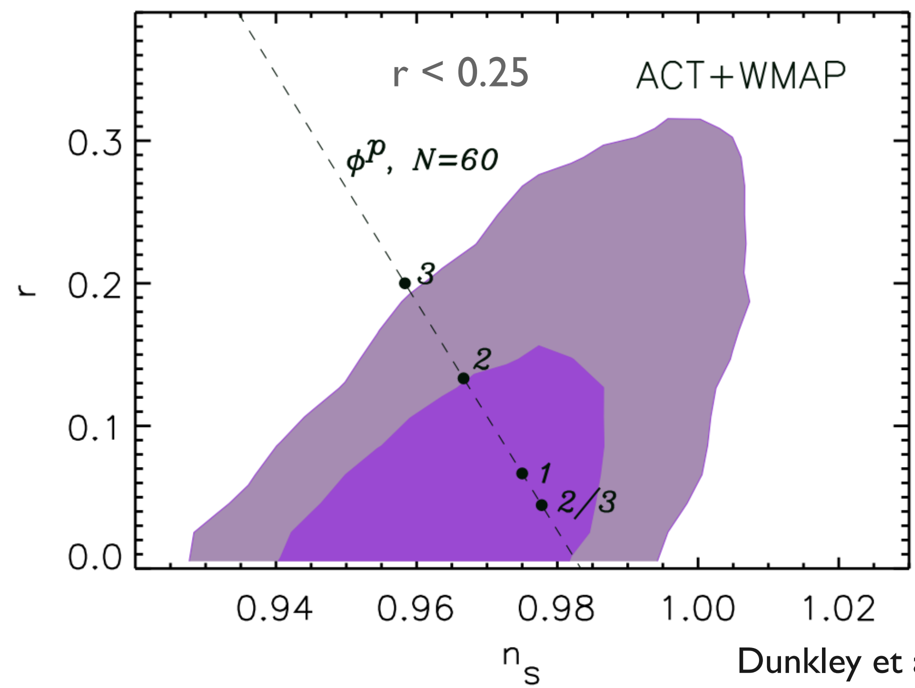
Dark Matter & Dark Energy



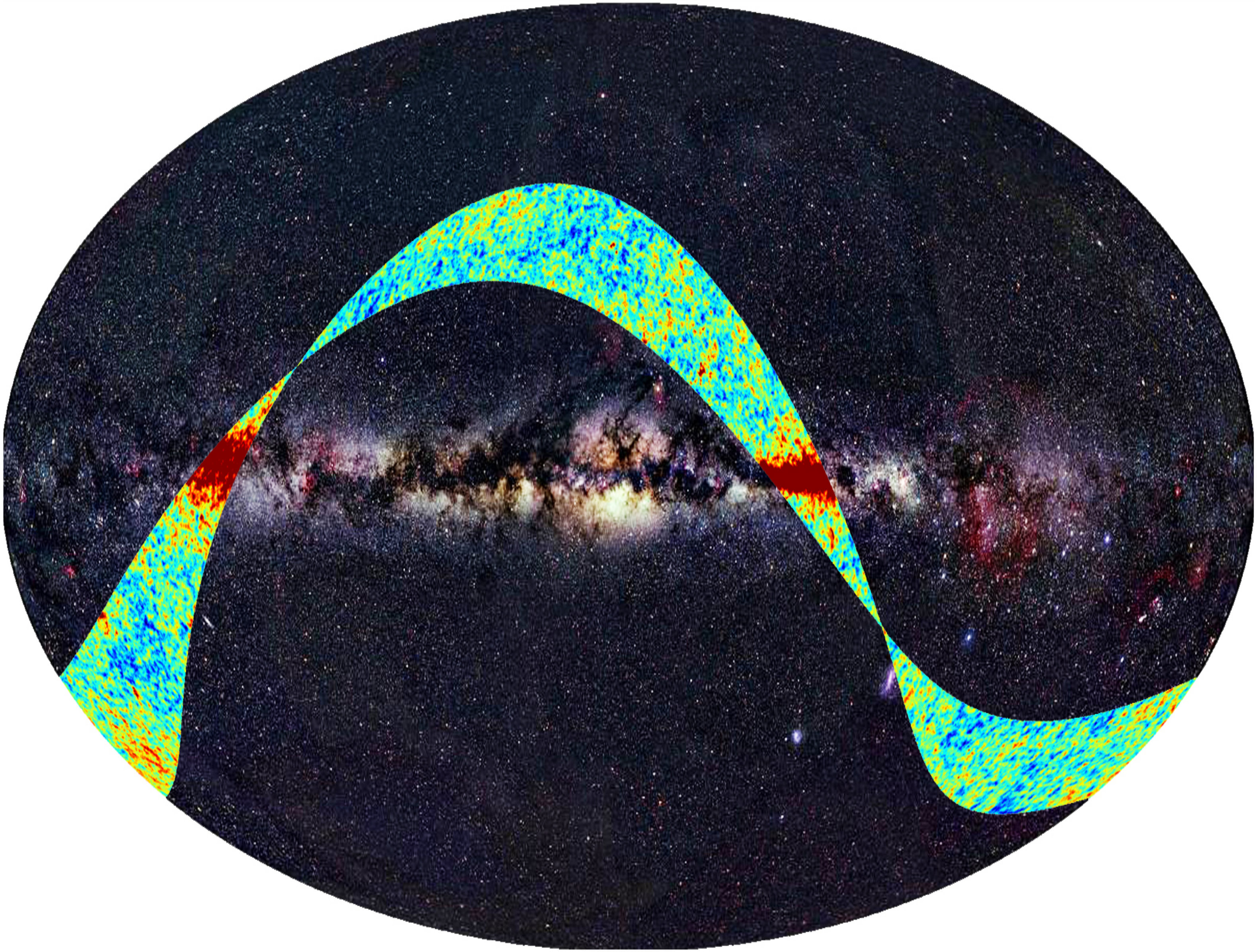
Inflation: limits from spectrum

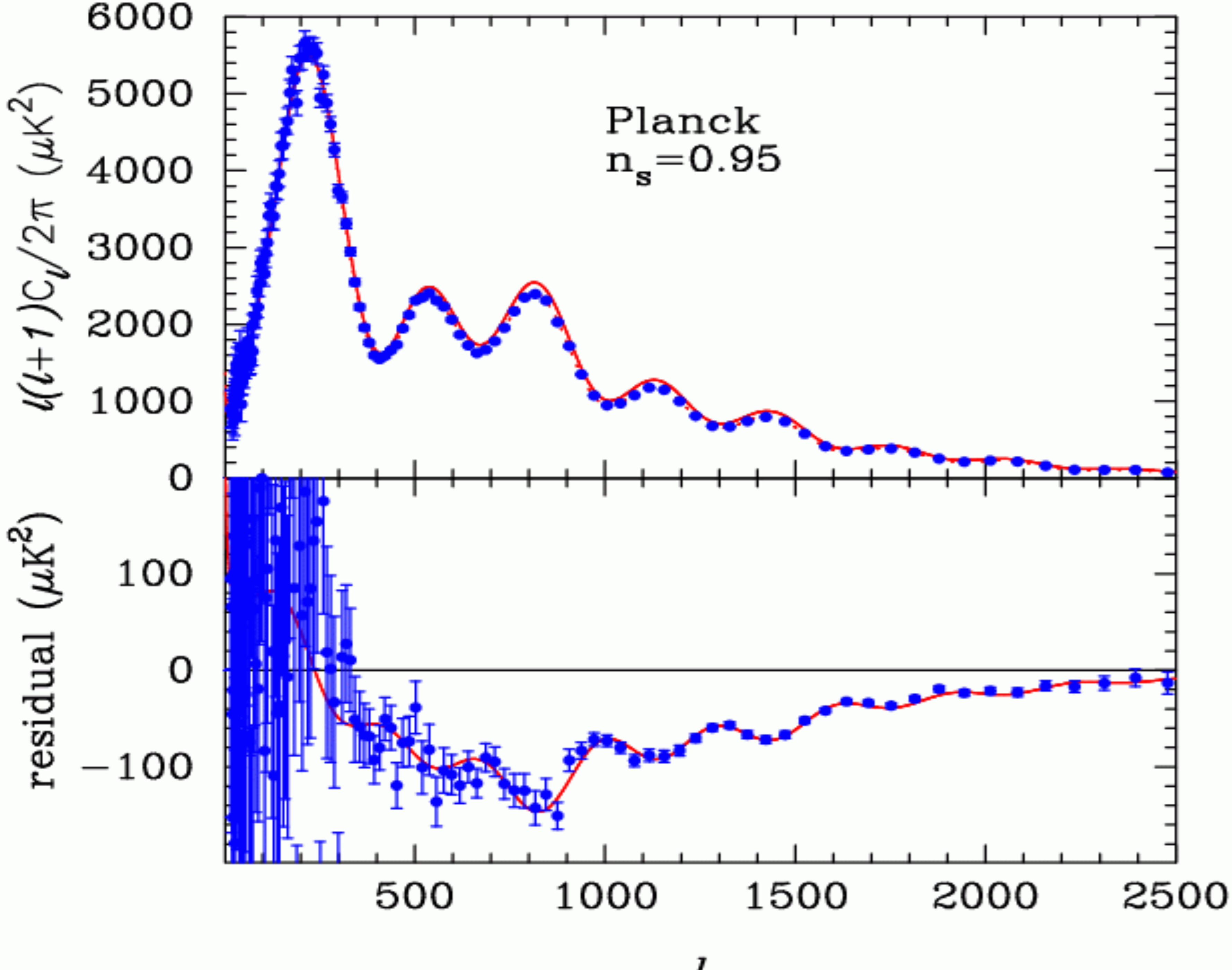
- All predictions so far consistent with CMB. Big goal: is index <1? The spectral index is still 3 sigma away. *Latest from WMAP + SPT*: n_s = 0.966 ± 0.011
  - Running index, find $dn_s/d\ln k$ = -0.024 ± 0.015 (ACT+WMAP+BAO+H0, similar with SPT)
    - New upper limit on tensors, find r < 0.19 (95% CL, ACT+WMAP+BAO+H0)



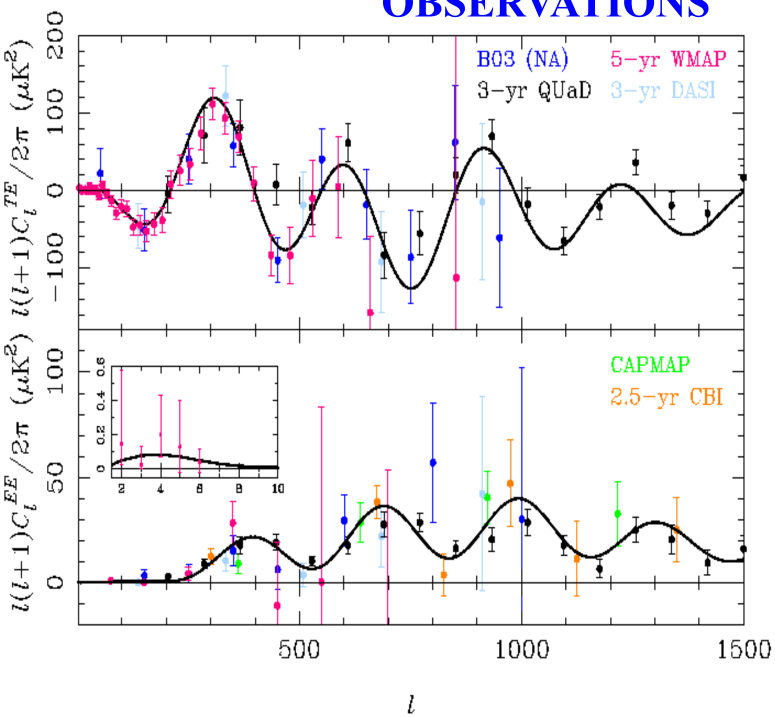


Dunkley et al. 2011

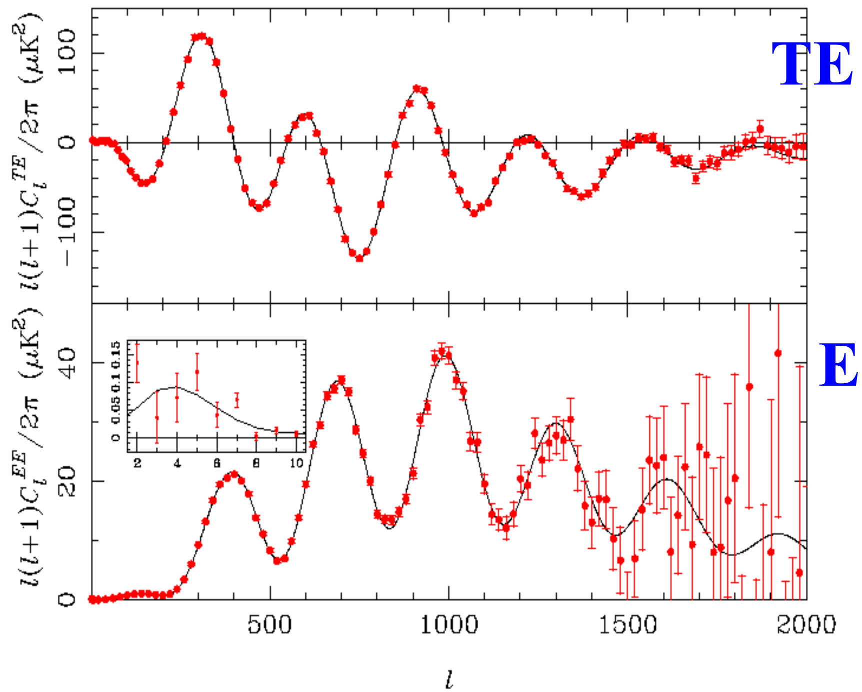



Planck
$n_s=0.95$
$\ell(\ell+1)C_\ell/2\pi\ (\mu K^2)$
residual (μK^2)
6000
5000
4000
3000
2000
1000
0
100
0
−100
500
1000
1500
2000
2500
l

CURRENT OBSERVATIONS



PLANCK FORECAST



"In models with the initial superdense de Sitter state ... such a large amount of relic gravitational waves is generated

...that ... the very existence of this state can be experimentally"

verified in the near future.

(Starobinsky, 1980)