Blackfolds

Fluid dynamics for hi-d black holes

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Motivation: GR as a tool

- GR is a theoretical ***tool*** – *much like QFT* – that can be applied in many areas of physics outside the traditional fields of astro/cosmo/…

  - Main recent developments from AdS/CFT:
    AdS/QCD, AdS/QGP, AdS/cond-mat, Fluid/Gravity…
  - And also String theory, TeV-gravity (bhs@colliders), etc

➔ Develop and understand better this tool

Motivation: GR as a tool

- Most basic set up: vacuum GR

$$R_{\mu\nu}=0$$

  - $\exists$ only one parameter for tuning: D

- Most basic objects: Black Holes

Motivation: GR as a tool

- Emphasis: instead of quick results with high-yield gain (applications), focus on developing fundamentals

  (*learn from financial crisis…*)

- When first found, black hole solutions have always been "answers waiting for a question"

Black Holes

- Main lesson from the "Golden Age" (1960's-70s):

  Black Holes are extremely simple objects, with very simple dynamics

- Main lesson from recent years:

  This simplicity occurs only in 4D.

  Hi-D black holes possess *qualitatively new* dynamics absent in 4D.

The trouble with Hi-D Black Hole Physics

- For several years we've been trying to reproduce the successful 4D programme:
  1. Find all black hole solutions (eg, of $R_{\mu\nu}=0)$ in closed analytic ("exact") form
  2. Classify them

- This seems now (to me)
  1. impossible (partial success in 5D, *hopeless* in D>5)
  2. maybe doable in low D (5, 6?...), but maybe not possible/useful/the right thing in higher D

A new framework

1. Identify why there is novel dynamics, different than 4D

2. Find new organizing framework

3. Develop new approaches to deal with it – new tools

A new framework

1. Identify why there is novel dynamics, different than 4D
   - Different length scales along the horizon
2. Find new organizing framework
   - Organize black holes according to scale hierarchy
3. Develop new approaches to deal with it – new tools
   - Effective theory at long wavelengths

Effective fluid in a dynamical worldvolume

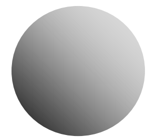
Black holes in 4D

- Kerr black holes $J \leqslant GM^2$

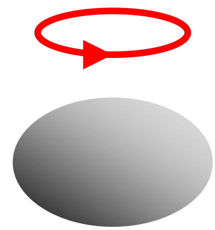
- Uniqueness theorem: *End of the story*

Black holes in *D*>4: known exact solutions

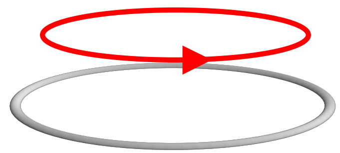
- Schwarzschild-Tangherlini in any D
  - much like Schwarzschild 4D
  - Unique static black hole, dynamically linearly stable



- Myers-Perry in any D
  - spherical topology
  - rotation in all possible planes



- Black ring in 5D
  - topology S^1x S^2
  - "circular black string"



4D vs hi-D Black Holes: Size matters

- Main novel feature of D>4 BHs: in some regimes they're characterized by two widely separate scales:

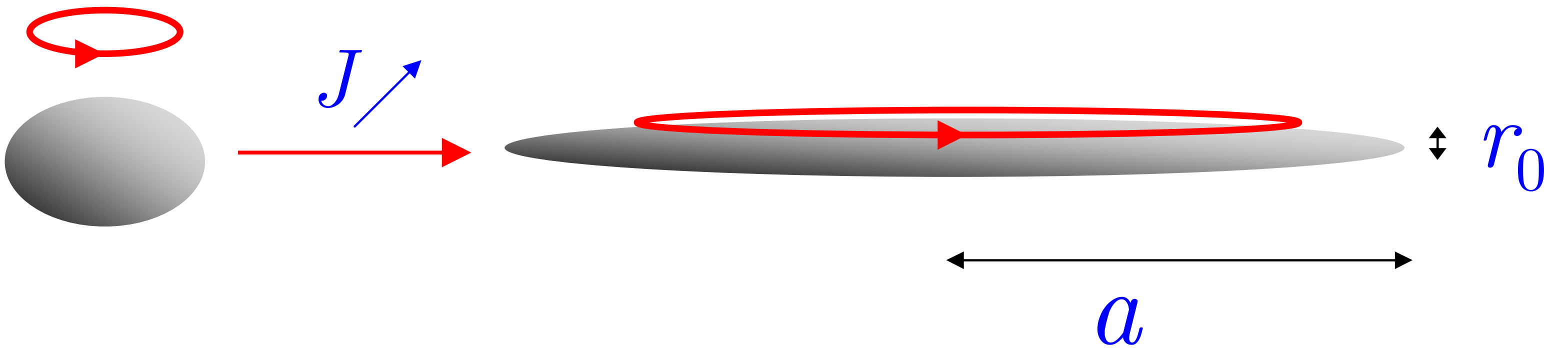
$$\ell_M \sim (GM)^{1/(D-3)}, \qquad \ell_J \sim J/M$$

- 4D BHs: single scale: $r_0 \sim GM$
  - no small parameter

- D>4 BHs: No upper bound on J for given M
  $\Rightarrow$ Length scales ℓ_M, ℓ_J can differ arbitrarily

Myers-Perry bhs in D⩾6:
Two scales and black brane limit

- Ultra-spinning regime $a \sim J/M \gg (GM)^{1/(D-3)}$

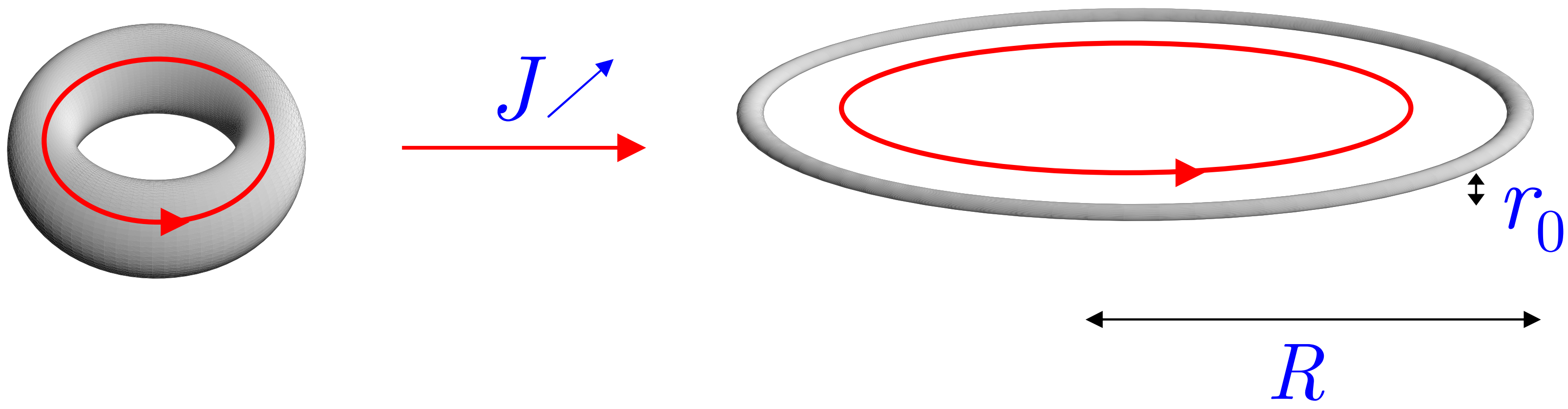


- Limit $a \to \infty$, r_0 finite:

$\Rightarrow$ black 2-brane along rotation plane

Black Ring in D=5
Two scales and black brane limit

- Ultra-spinning regime $R \sim J/M \gg (GM)^{1/(D-3)}$

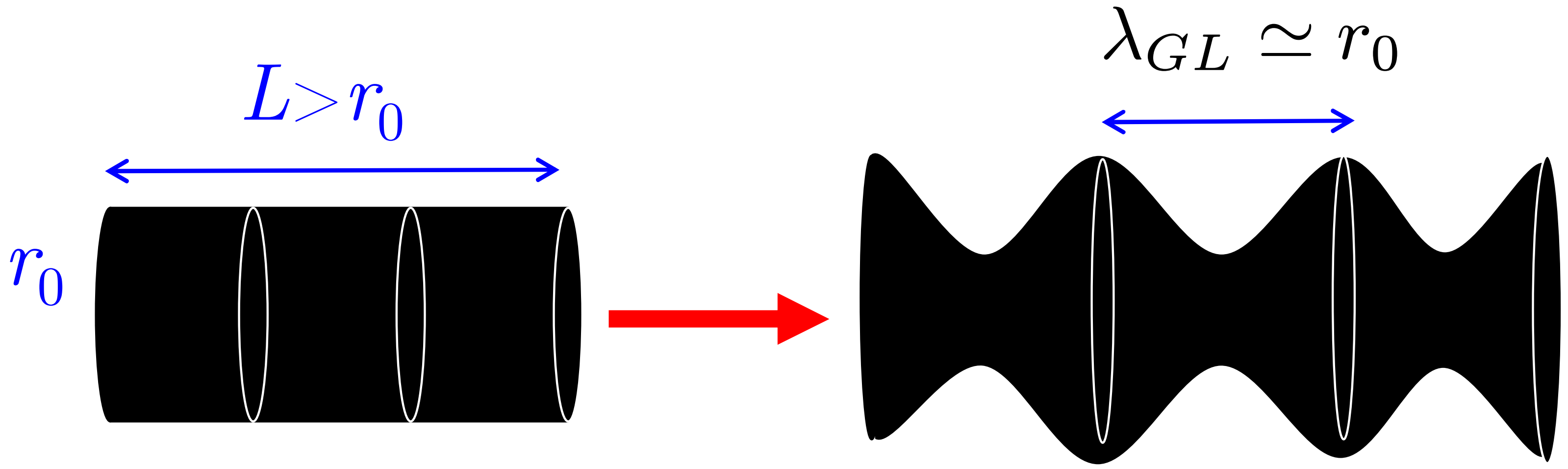


- Limit $R \to \infty$, r_0 finite:

$\Rightarrow$ black string along rotation direction

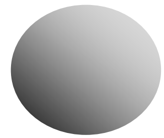
Note also:

- Gregory-Laflamme instability of black brane when the two scales $r_0, \ L$ begin to differ



Organization in scales

- $\ell_J \lesssim \ell_M$



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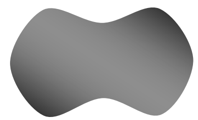
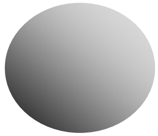


- $\ell_J \gg \ell_M$

Organization in scales

- $\ell_J \lesssim \ell_M$: single scale, Kerr-like – not much new expected: uniqueness, stability



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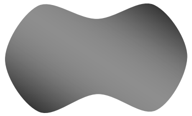

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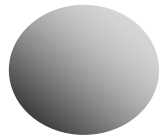
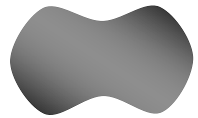


- $\ell_J \sim \ell_M$: threshold of separating scales: GL-like instabilities, inhomogeneous ("pinched") phases, mergers – most difficult to study analytically, but better for numerics



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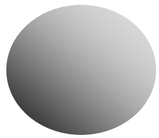

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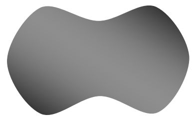


- $\ell_J \gg \ell_M$: separated scales. Natural approach: integrate out short-distance physics, find long-distance effective theory

Effective theory at large length scales

- Separate long- and short-wavelength d.o.f.'s
- Replace short-distance d.o.f.'s with effective theory

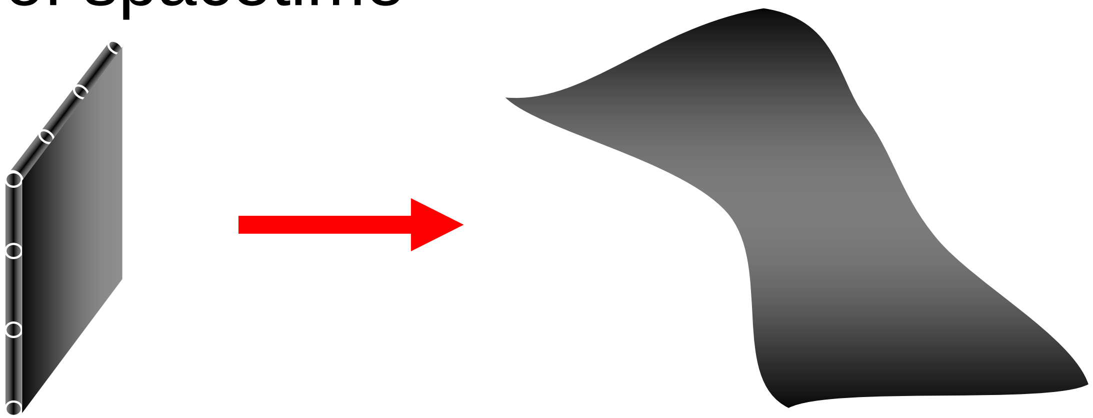
$$I_{\rm EH} = \int \sqrt{-g} R \approx \int_{\lambda \gg r_0} \sqrt{-g_{(\rm long)}} R_{(\rm long)} + I_{\rm eff}[g_{(\rm long)}, \phi(\sigma)]$$

- What kind of effective theory?
  - Hint: limit $\ell_M/\ell_J \to 0$ yields a black brane

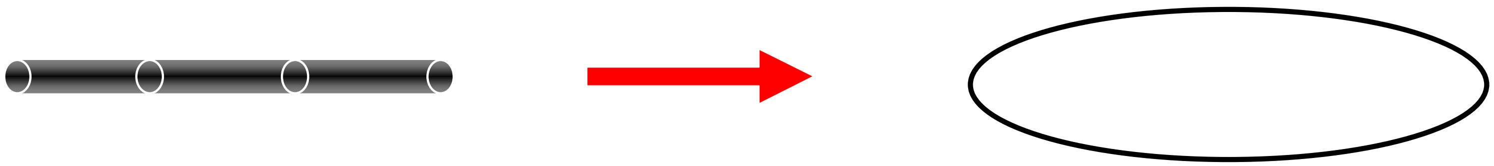
$\Rightarrow I_{\rm eff}$ is a worldvolume theory for the "collective coordinates" $\phi(\sigma)$ of a black brane

Black holes from *blackfolds*

- **Blackfold**: **Black** p-brane w/ worldvolume = curved submani**fold** of spacetime



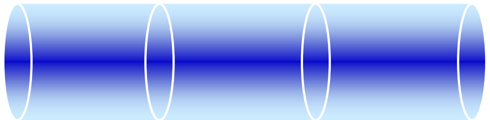
- If blackfold worldvolume is spatially compact, then it describes a black hole
  - Eg, black ring as circular black string:



Analogous effective theories

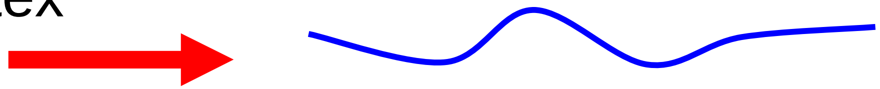
- Cosmic strings from Nielsen-Olesen vortices

Abelian Higgs model



N-O vortex

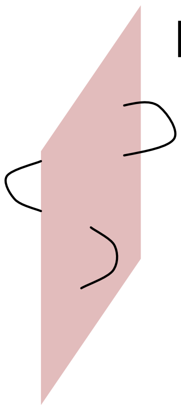
Nambu-Goto string



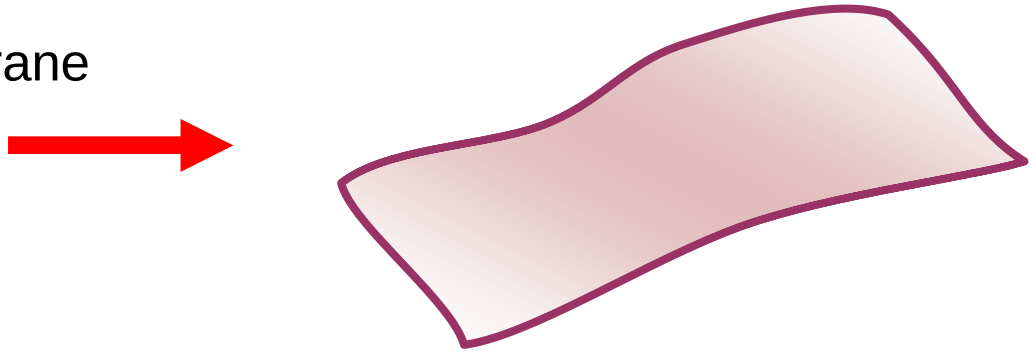
Collective coords: $X^\mu(\sigma^\alpha)$

- D-branes in string theory

Dirichlet-brane

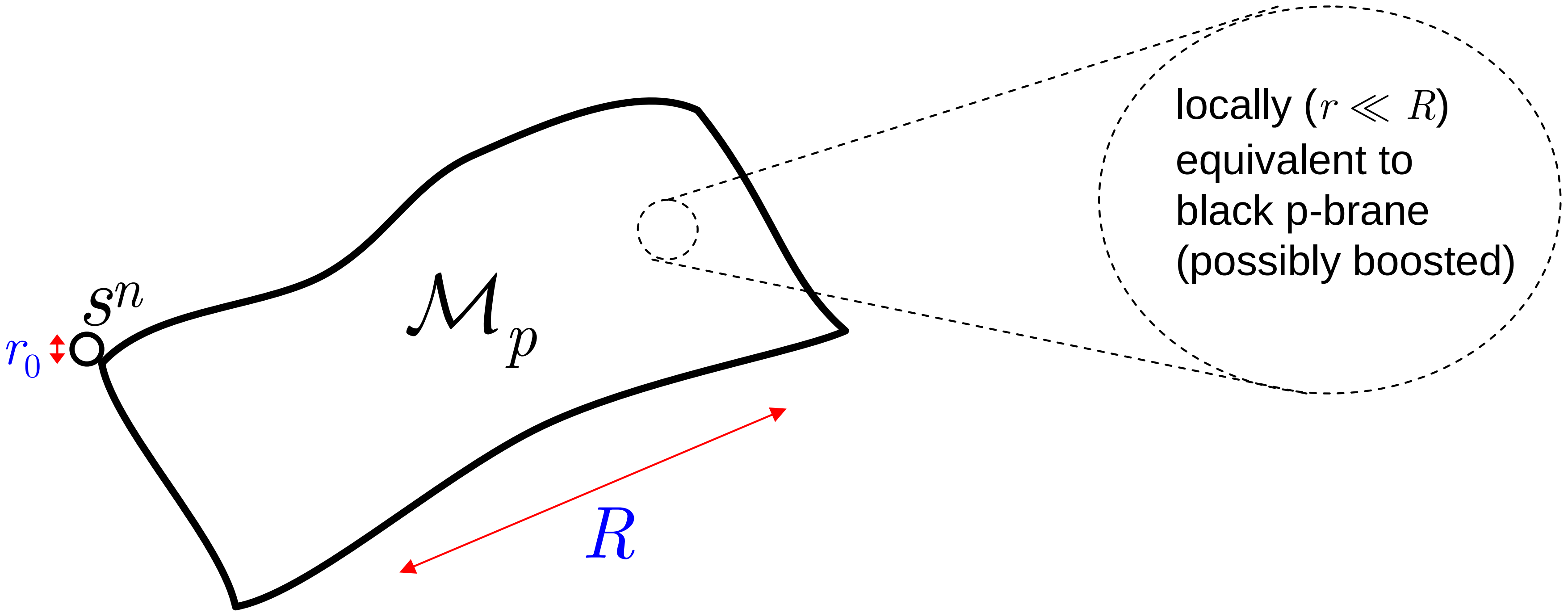


Dirac-Born-Infeld brane



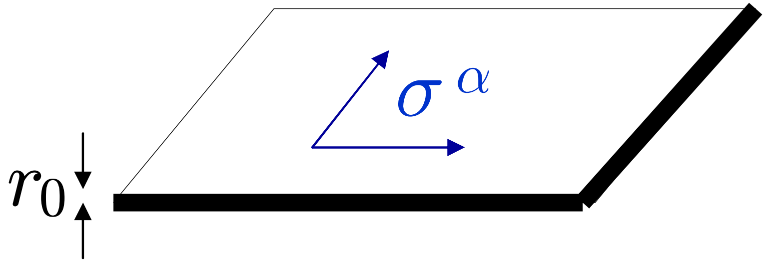
Collective coords: $X^\mu(\sigma^\alpha),\ A^\mu(\sigma^\alpha)$

Blackfolds: long-distance effective dynamics of hi-d black holes



- Black p-brane (w/ velocity u^α, $u^\alpha u_\alpha = -1$)

$$ds^2 = \left(\eta_{\alpha\beta} + \frac{r_0^n}{r^n} u_\alpha u_\beta\right) d\sigma^\alpha d\sigma^\beta + \frac{dr^2}{1 - \frac{r_0^n}{r^n}} + r^2 d\Omega_{n+1}^2$$

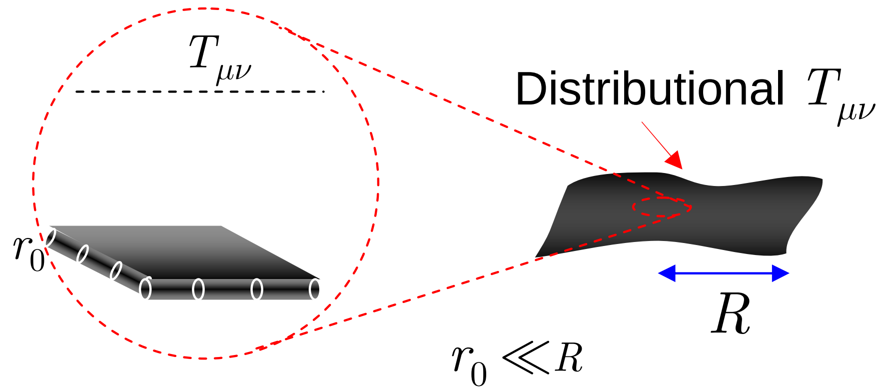


- Collective coordinates:
  - 'thickness' $r_0(\sigma)$,
  - velocity $u^\alpha(\sigma)$,
  - transverse coords $X^\mu(\sigma)$

- Long-wavelength variations: $r_0 \nabla_\alpha \ll 1$

- Effective stress tensor (at long distance)

$$T_{\alpha\beta} = r_0^n (n u_\alpha u_\beta - \eta_{\alpha\beta})$$



This is an effective perfect fluid:

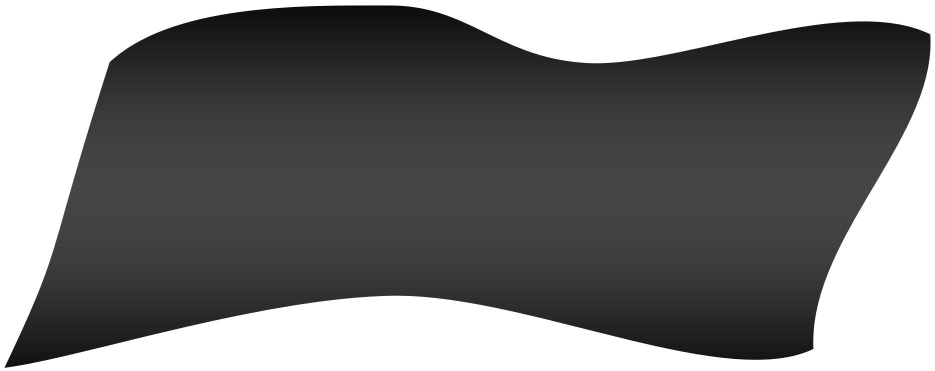
$$\varepsilon = (n+1) {r_0}^n \,, \quad P = -{r_0}^n$$

- Now, find equations for the collective field variables

General Classical Brane Dynamics

- Equtions for any worldvolume source of energy-momentum, in probe (test brane) approx,

$T_{\mu\nu}$



$$\nabla_{\mu} T^{\mu\nu} = 0$$

General Classical Brane Dynamics

- To lowest gradient-order $T_{\alpha\beta} =$ (perfect fluid)
- Along **worldvolume** directions:

$$\nabla_\alpha T^{\alpha\beta} = 0$$

$\Rightarrow$ **Worldvolume Fluid** equations (Euler)

- Along **transverse** directions:

Carter

$$\nabla_\mu T^{\mu\rho} = 0 \quad \Rightarrow \quad T^{\mu\nu} K_{\mu\nu}{}^{\rho} = 0$$

extrinsic curvature

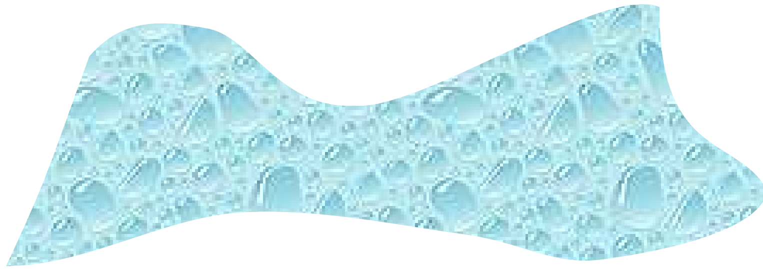
$\Rightarrow$ **Generalized geodesic** equations ("mass x acceleration = 0")

General Classical Brane Dynamics

- Classical brane dynamics is the **dynamics of a fluid on a dynamical worldvolume**

  Fluid-dyn is effective description

  of worldvolume thy at long wavelengths



- For the ‘black brane fluid’ $\varepsilon = (n+1){r_0}^n \ , \ P = -{r_0}^n$

$$\dot{u}^\mu + \frac{1}{n+1} u^\mu \overline{\nabla}_\nu u^\nu = \frac{1}{n} K^\mu + \overline{\nabla}^\mu \ln r_0$$

Blackfold equations

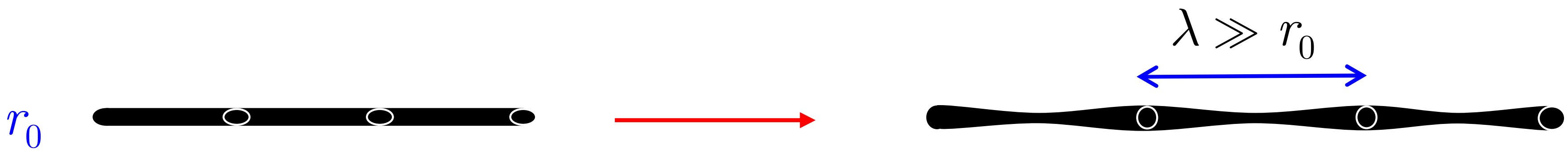
Black hole dynamics as fluid dynamics

- Not too surprising: Fluid dynamics is the *most general* effective theory for long-wavelength fluctuations off thermo equil
- The Fluid/Gravity correspondence *(Bhattacharya+Hubeny+Minwalla+Rangamani)* for black holes in AdS: particular case of blackfold dynamics
  - Fluid is near-extremal D3-brane
  - Worldvolume is not dynamical
- Broader viewpoint: connect to “membrane paradigm”, deeper understanding of “black holes as fluids”

Gregory-Laflamme as sound-mode instability

- Consider pressure (sound) waves of the worldvolume "black brane fluid":

$$P = -{r_0}^n \ , \quad \delta P \longrightarrow \delta r_0$$



- Sound velocity $\quad {c_s}^2 = \mathrm{d}P/\mathrm{d}\varepsilon < 0$: unstable

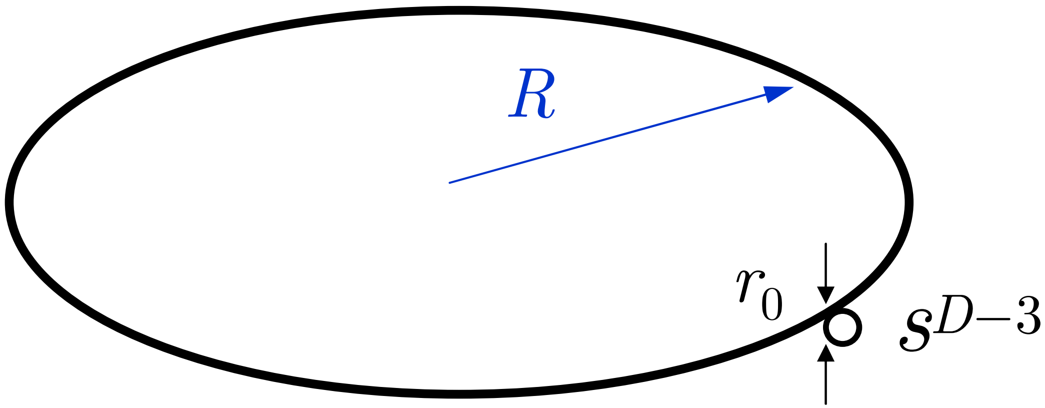
long-wavelength tail of GL-instability

- GL = sound-mode instability in the effective fluid

The Blackfold Gallery

- Simplest example: black rings in $D \geqslant 5$

$D = n + 4$

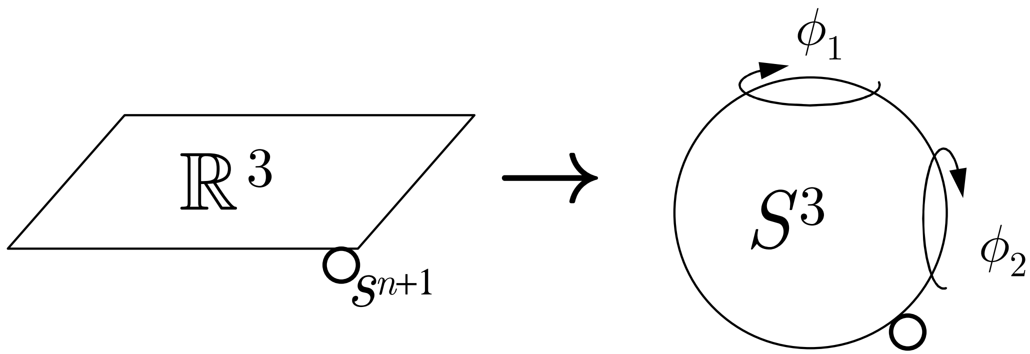


$\longrightarrow$ black holes with horizon S^1 x s^{D-3}

"small" transverse sphere $\sim r_0$

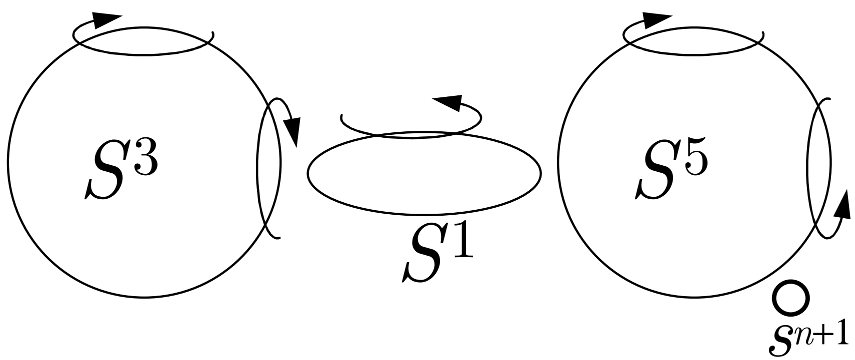
- Products of spheres

- $\mathbb{R}^3 \, \mathrm{x} \, s^{n+1} \rightarrow S^3 \, \mathrm{x} \, s^{n+1}$



- Can do it for any product of odd-spheres

$$\prod_{p_a \in \mathrm{odd}} S^{p_a} \times s^{n+1}$$



• Solving a conjecture on horizon symmetries

- Rigidity of horizons: How many spatial $U(1)$ isometries must a bh horizon have?
- *Hollands+Ishibashi+Wald* : at least one

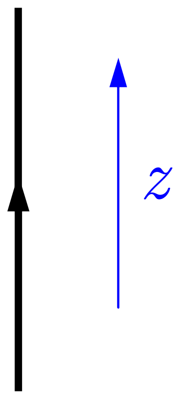
$$\partial_t + \Omega \partial_\phi$$

- But *possibly more*: MP bhs and black rings have all the Cartan subgroup of $O(D-1) \supset U(1)^{\lfloor (D-1)/2 \rfloor}$
  - e.g. 5D bhs have spatial isometry $U(1)_{\phi_1} \times U(1)_{\phi_2}$
- *Reall* conj. (2002): $\exists$ hi-d bhs w/ only $U(1)_\phi$

The solution: Helical blackfolds

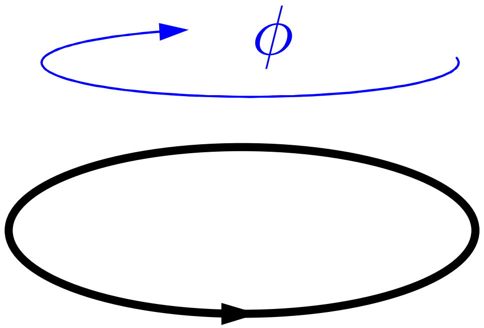
- Place a boosted black string along an isometry ζ of background

$$\zeta = \partial_z$$



black string

$$\zeta = \partial_\phi$$



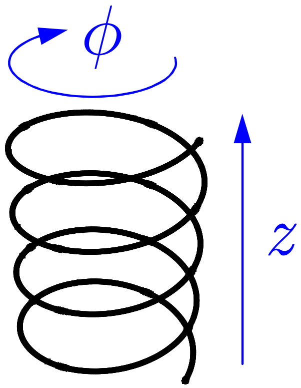
black ring

The solution: Helical blackfolds

- Place a boosted black string along an isometry ζ of background

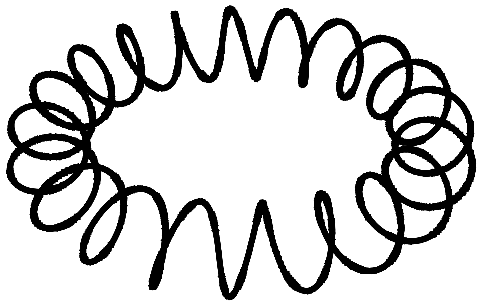
$$\zeta = k\partial_z + \partial_\phi$$

Helical
black string



$$\zeta = n\partial_{\phi_1} + m\partial_{\phi_2}$$

Helical
black ring
(*slinky*)

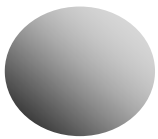


(n.b: profile is static!)

- The orthogonal isometry is broken:
  Horizon has *only one* spatial U(1)

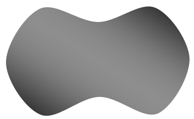
Conclusions

- We don't know the landscape of hi-d bhs in detail yet, but now **we have a map**
- Black hole dynamics splits into three regimes according to size hierarchies

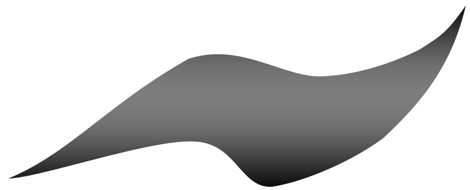
1: 

Solved?

4D-like

2: 

Numerics (making progress)

3: 

Blackfolds: we have the theory

No 4D counterpart

Conclusions

- Hi-D black hole dynamics (at large spin) is
  - brane-like: elastic dynamics
  - fluid-like: hydrodynamics
- New dynamics → change focus
  - less emphasis on exact solutions
  - get used to approximate effective descriptions
- **Classification?**
  - Close to completing it in 5D
  - But it becomes increasingly harder at higher D