Multicenter non-BPS Black Holes

and what can they teach us about the Information Paradox

Iosif Bena

IPhT (SPhT), CEA Saclay

with Nick Warner, Gianguido Dall'Agata, Clement Ruef, Stefano Giusto

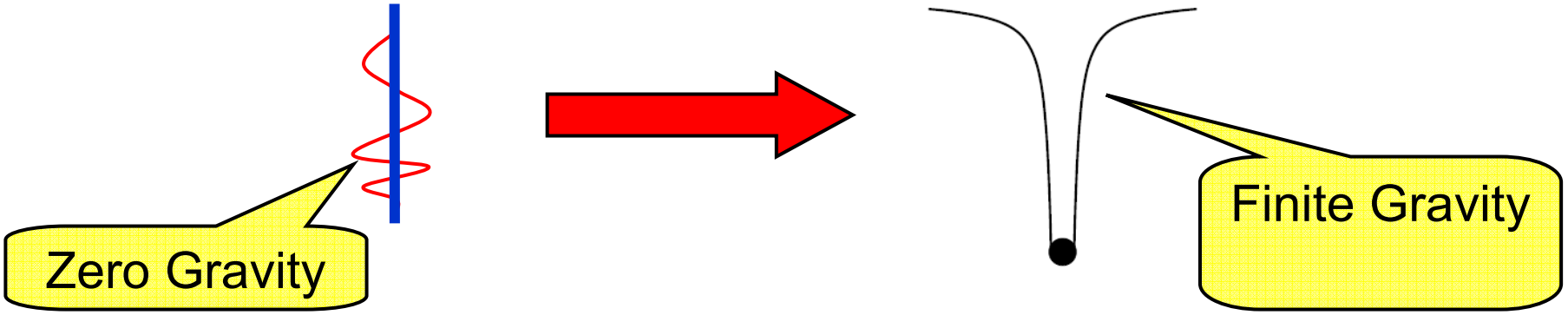
Strominger and Vafa (1996)

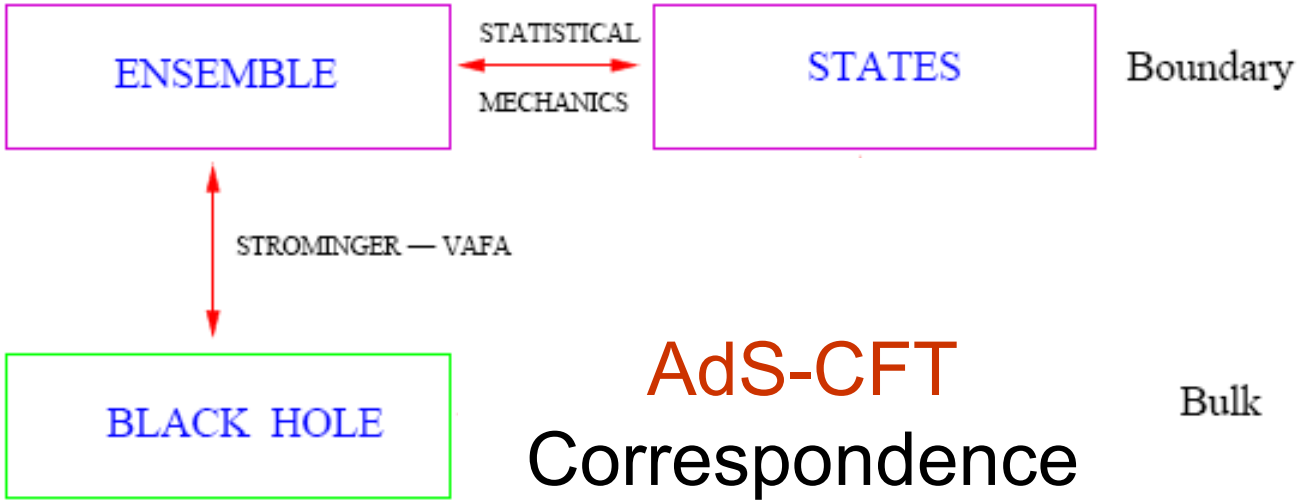
+1000 other articles

Count BH Microstates

Match B.H. entropy !!!

2 ways to understand:





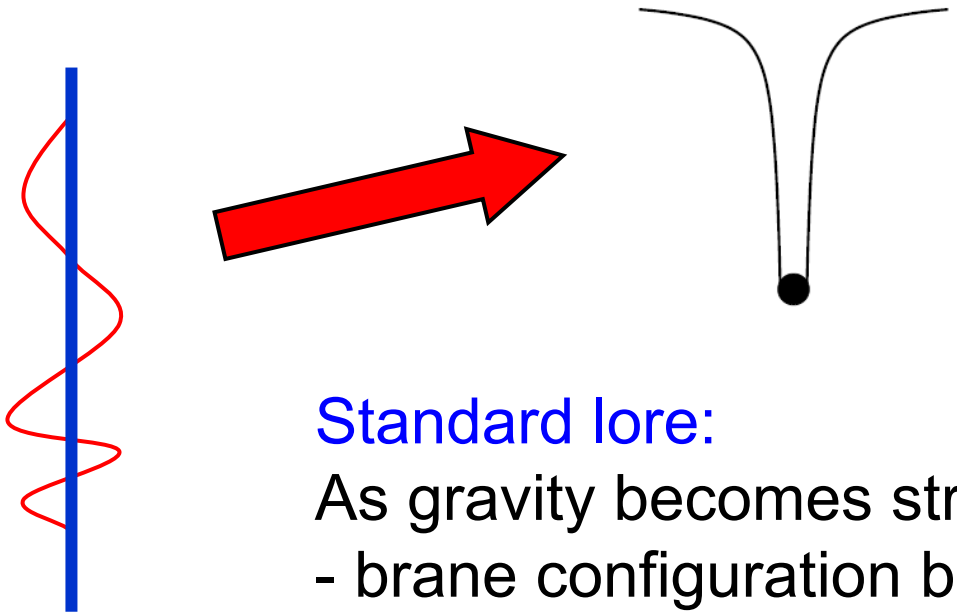
Strominger and Vafa (1996):

Count Black Hole Microstates (branes + strings)

Correctly match B.H. entropy !!!

Zero Gravity

Black hole regime of parameters:



Standard lore:

As gravity becomes stronger,

- brane configuration becomes smaller
- horizon develops and engulfs it
- recover standard black hole

Susskind
Horowitz, Polchinski
Damour, Veneziano

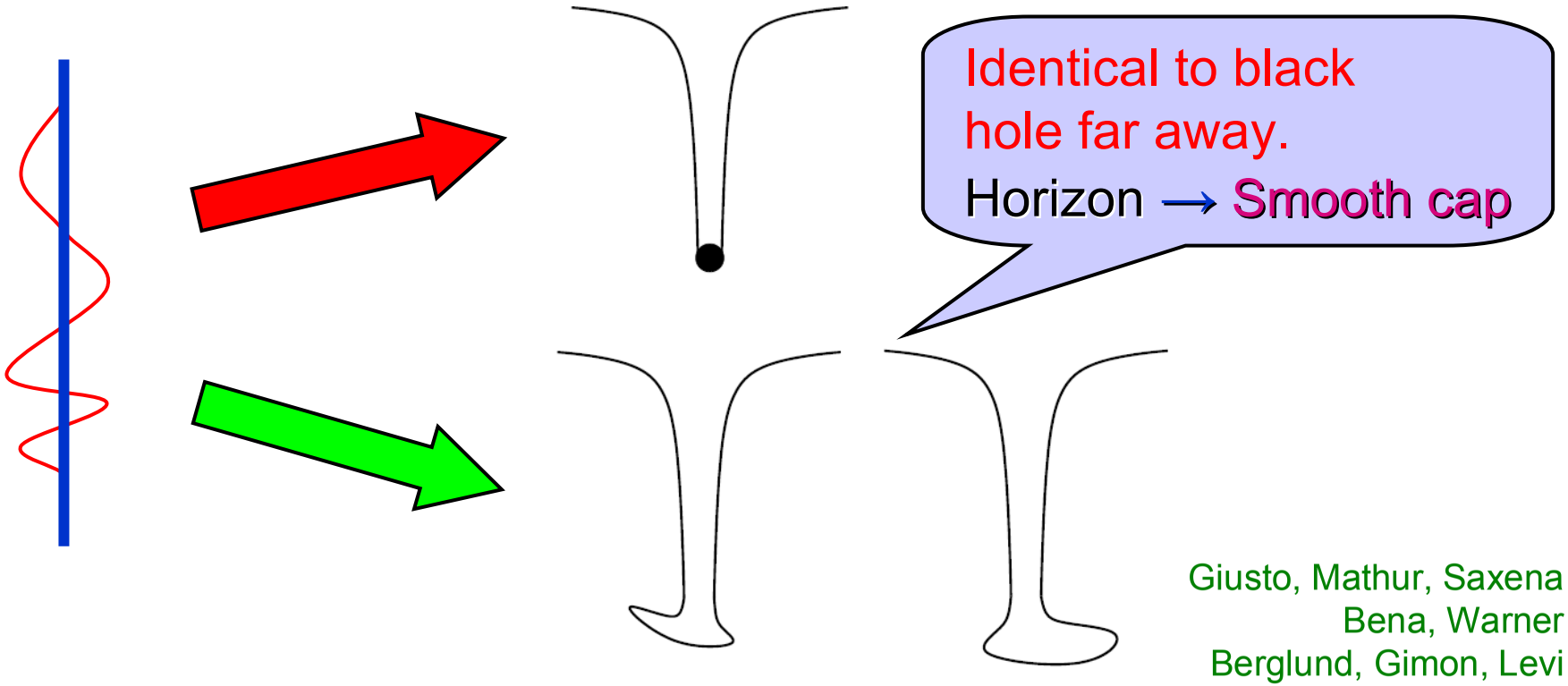
Strominger and Vafa (1996):

Count Black Hole Microstates (branes + strings)

Correctly match B.H. entropy !!!

Zero Gravity

Black hole regime of parameters:



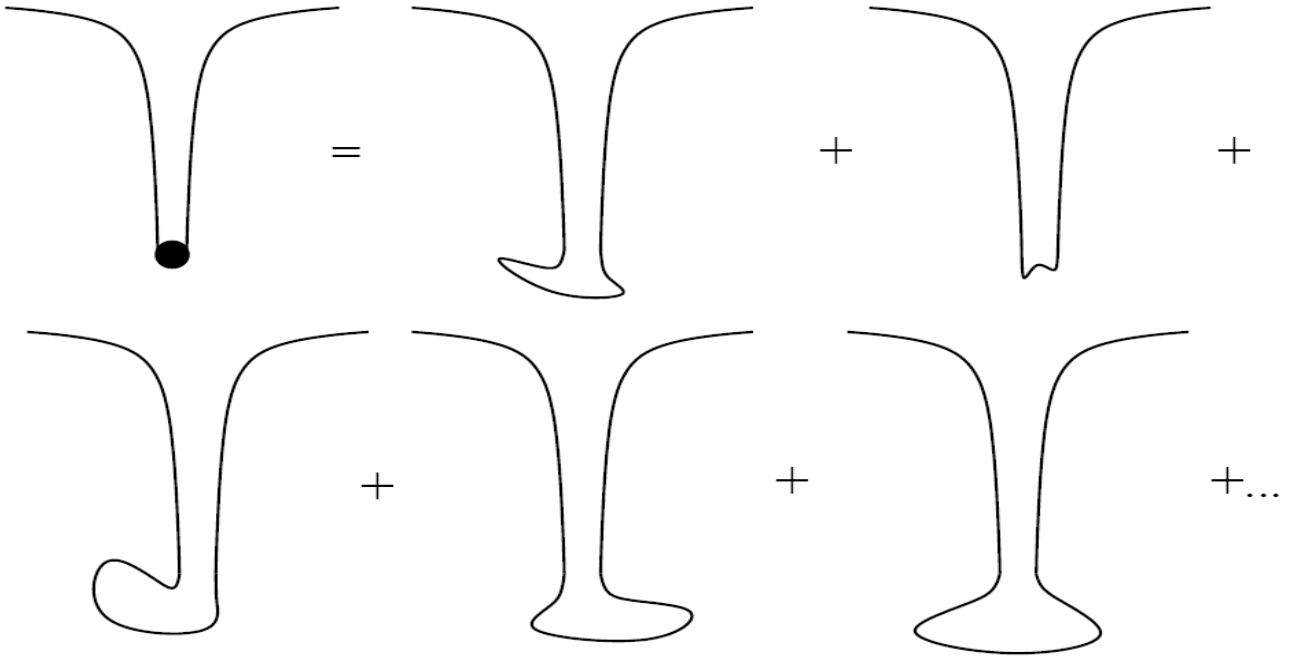
Giusto, Mathur, Saxena
Bena, Warner
Berglund, Gimon, Levi

BIG QUESTION: Are ***all*** black hole microstates becoming geometries with no horizon ?

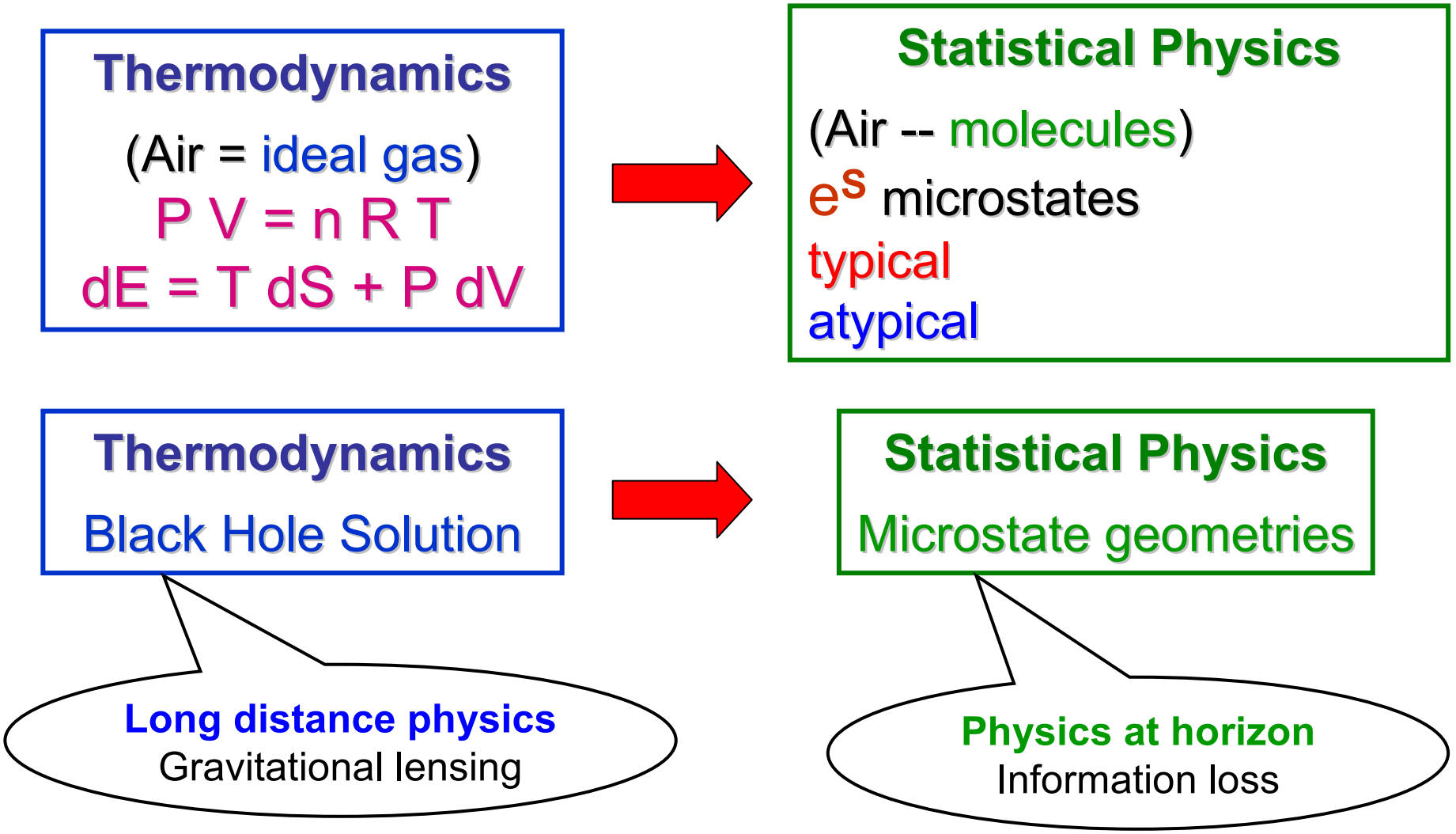
?

Black hole = ensemble of horizonless microstates

Mathur & friends



Analogy with ideal gas



A few corollaires

new low-mass degrees of freedom

- Thermodynamics (LQFT) breaks down at horizon. Nonlocal effects take over.

- No spacetime inside black holes. Quantum superposition of microstate geometries.

Can be proved by rigorous calculations:

1. Build most generic microstates + Count
2. Use *AdS*-CFT

∞ parameters
black hole charges

Word of caution

- To replace classical BH by BH-sized object
  - Gravastar
  - Fuzzball
  - LQG muck
  - Quark-star, you name it …

satisfy very stringent (mutilating) test: Horowitz

Same growth with G_N !!!

- BH size grows with G_N
- Size of objects in other theories becomes smaller

- BH microstate geometries pass this test
- Highly nontrivial mechanism

BPS Microstates geometries

M2	0	1	2						
M2	0			3	4				
M2	0					5	6		

3-charge 5D black hole Strominger, Vafa; BMPV

$$S_{BMPV} = 2\pi\sqrt{N_1 N_5 N_P - J^2}$$

$$ds^2 = Z_1^{-2/3} Z_2^{-2/3} Z_3^{-2/3} (dt + \vec{k})^2 + Z_1^{1/3} Z_2^{1/3} Z_3^{1/3} dx^2_{\mathbb{R}^4} + ds^2_{T^6}$$

$$F_{120i} = \partial_i Z_1^{-1} \qquad F_{340i} = \partial_i Z_2^{-1} \qquad F_{560i} = \partial_i Z_3^{-1} \qquad \text{electric}$$

Want solutions with same asymptotics, but no horizon

BPS Microstates geometries

M2	0	1	2						
M2	0			3	4				
M2	0					5	6		
M5	0			3	4	5	6		θ
M5	0	1	2			5	6		θ
M5	0	1	2	3	4				θ

CLOSED CURVE

T^6 $\mathbb{R}^4$

$$ds^2 = Z_1^{-2/3} Z_2^{-2/3} Z_3^{-2/3} (dt + \vec{k})^2 + Z_1^{1/3} Z_2^{1/3} Z_3^{1/3} dx^2_{\mathbb{R}^4} + ds^2_{T^6}$$

$$F_{120i} = \partial_i Z_1^{-1} \qquad F_{340i} = \partial_i Z_2^{-1} \qquad F_{560i} = \partial_i Z_3^{-1} \qquad \text{electric}$$

$$F_{12ij} = G^1_{ij} \qquad F_{56ij} = G^2_{ij} \qquad F_{56ij} = G^3_{ij} \qquad \text{magnetic}$$

Solution depends on G^1 G^2 G^3 Z_1 Z_2 Z_3 $\vec{k}$

Bena, Warner
Gutowski, Reall

BPS Microstates geometries

Linear system

4 layers:

Bena, Warner

$\mathbb{R}^4$ base (4D Hyper Kahler)

$$*G^I = G^I$$

$$d * dZ_1 = G^2 \wedge G^3$$

$$d\vec{k} + *d\vec{k} = G^1 Z_1 + G^2 Z_2 + G^3 Z_3$$

M2 Killing Spinors !!!

Focus on Gibbons-Hawking (Taub-NUT) base:

$$ds^2 = V\left(dx_1^2 + dx_2^2 + dx_3^2\right) + V^{-1}(d\psi + \vec{A})^2$$

$$\nabla \times \vec{A} = \nabla V$$

$V = \frac{1}{r}$ $\mathbb{R}^4$

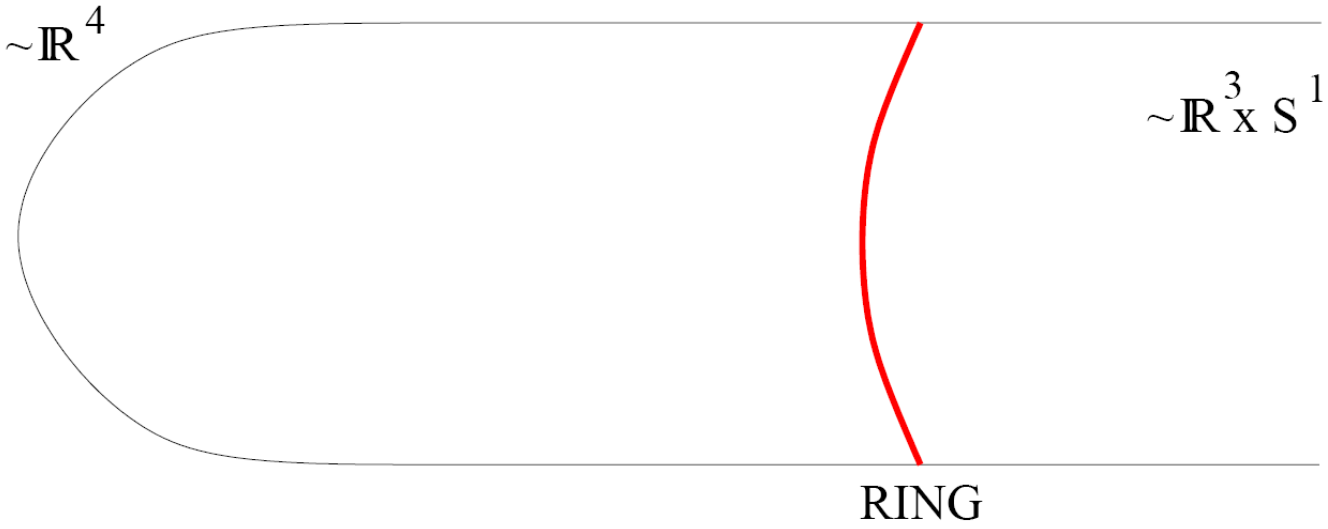
$V = 1 + \frac{1}{r}$ Taub-NUT

8 harmonic functions

Gauntlett, Gutowski,
Bena, Kraus, Warner

BPS Black Rings (in Taub-NUT)

Elvang, Emparan, Mateos, Reall; Bena, Kraus, Warner; Gaiotto, Strominger, Yin



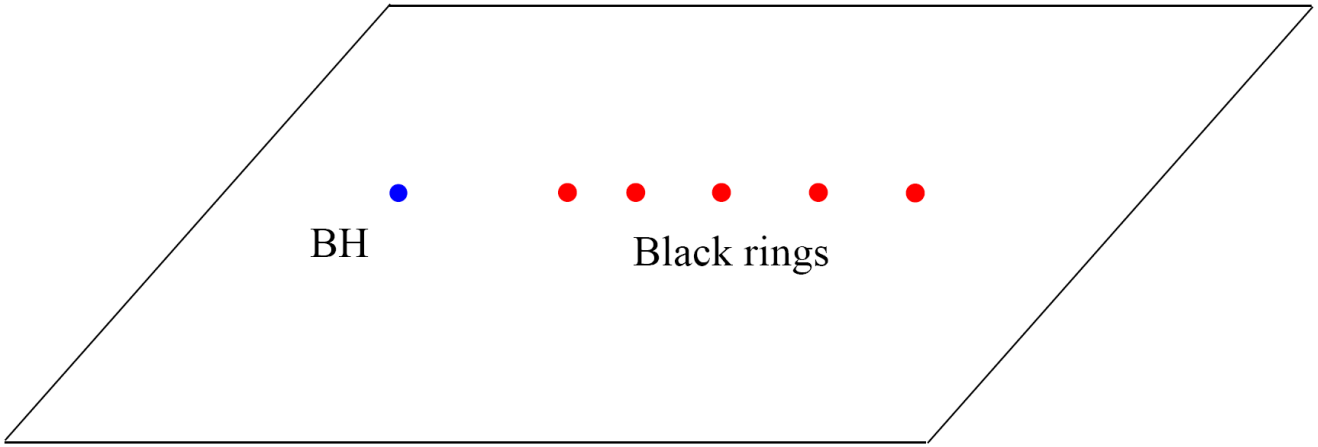
$$S = \pi\sqrt{2n_1n_2\bar{N}_1\bar{N}_2 + 2n_1n_3\bar{N}_1\bar{N}_3 + 2n_2n_3\bar{N}_2\bar{N}_3 - n_1^2\bar{N}_1^2 - n_2^2\bar{N}_2^2 - n_3^2\bar{N}_3^2 - 4n_1n_2n_3J_T}$$

4D BH: D2 charges $\bar{N}_1\ \bar{N}_2\ \bar{N}_3$, D4 charges $n_1\ n_2\ n_3$ and D0 charge J_T

- Position of ring depends on charges and moduli
- Ring can go to infinity and disappear from spectrum
- Lines of marginal stability, wall crossing, and all that …

Examples: Multiple Black Rings

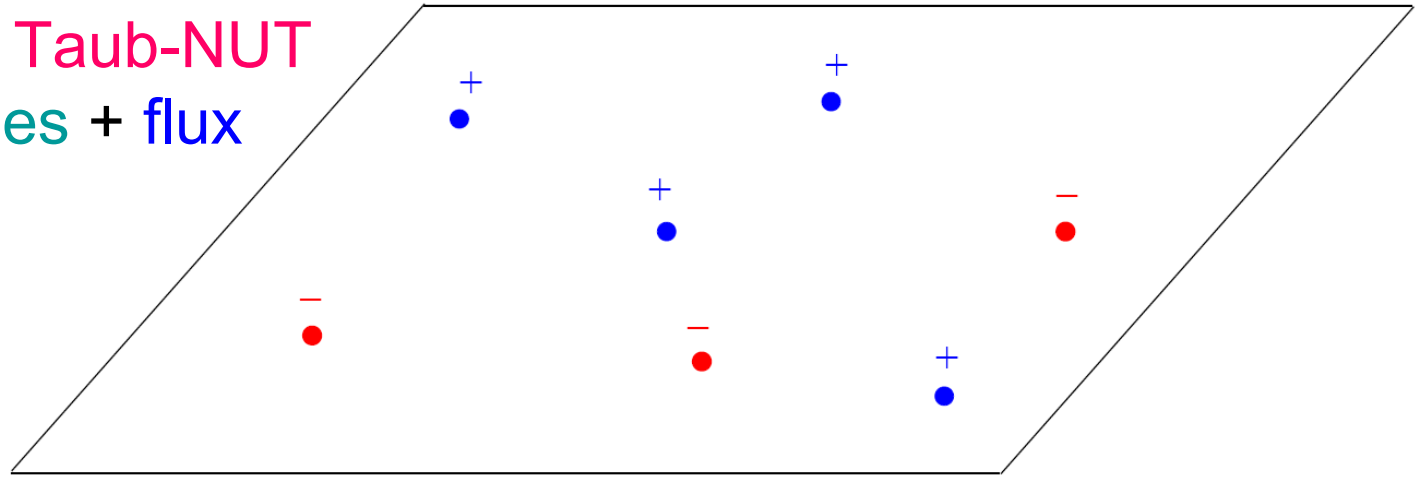
- 5D BH on tip of Taub-NUT ⇔ 4D BH with D6 charge
- Black ring with BH in the middle ⇔ 2-centered 4D BH
- 17 black rings + BH ⇔ 18-centered 4D BH Denef



- 4D D6,D4,D2,D0 BH ⇔ 5D black hole
- 4D D4,D2,D0 BH ⇔ 5D black ring
- 5D: ring supported by angular momentum
- 4D: multicenter configuration supported by ***E x B***

Microstates geometries

Multi-center Taub-NUT
many 2-cycles + flux



Compactified to 4D → multicenter configuration Denef

- $+$ GH center $\Leftrightarrow$ $\mathrm{D6}$ brane
- $-$ GH center $\Leftrightarrow$ $\overline{\mathrm{D6}}$ brane

Abelian worldvolume flux
Each: 16 supercharges
4 common supercharges
(D2,D2,D2)

Microstates geometries

- Where is the BH charge ?

$\mathbf{L} = q\, A_0$ magnetic

$\mathbf{L} = \ldots + A_0\, F_{12}\, F_{34} + \ldots$

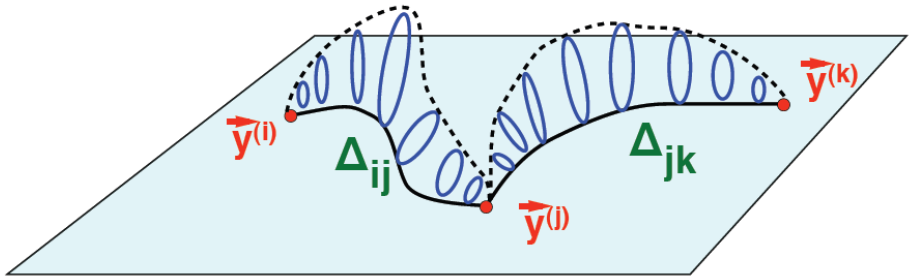
- Where is the BH mass ?

$\mathbf{E} = \ldots + F_{12}\, F^{12} + \ldots$

- BH angular momentum

$\mathbf{J} = E \times B = \ldots + F_{01}\, F_{12} + \ldots$

2-cycles + magnetic flux

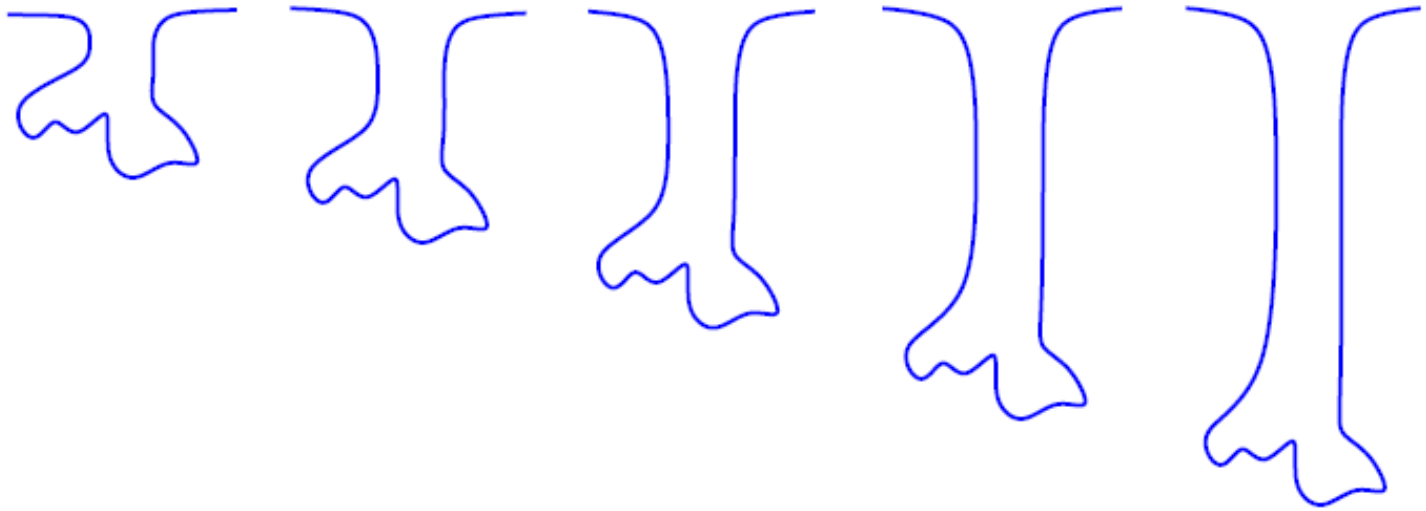


Charge disolved in fluxes

Klebanov-Strassler

$$\int_{a-b} F_{12ij} = n_1 \qquad \int_{O-a} F_{34ij} = f_2$$

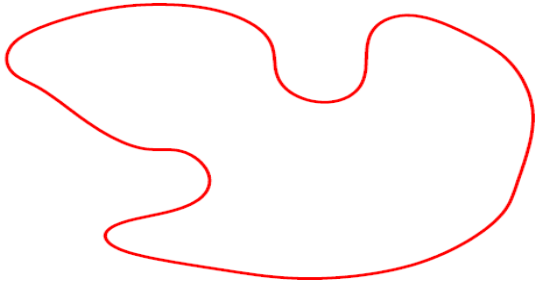
The deep microstates



- 4D perspective: points collapse on top of each other
- 5D: throat deeper and deeper; cap remains similar !
- Solution smooth throughout scaling !
- Long throats → small mass gap → typical CFT sector
- Scaling goes on forever !!! AdS-CFT unhappy
  - Can it be stopped ? Quantum effects ? see Jan's talk
  - Destroy huge chunk of a smooth horizonless solution ?

More general solutions

- Put supertube in bubbling solution
- Supertubes Mateos, Townsend; Emparan
  - supersymmetric brane configs.
  - arbitrary shape:
  - smooth supergravity solutions
    Lunin, Mathur; Lunin, Maldacena, Maoz



- Classical moduli space of microstates solutions has infinite dimension !
  - Much bigger than space Jan counts (extra U(1))
  - Key ingredient in getting correct D1-D5 entropy
  - Wiggly supertubes do not descend to 4D sugra.

More general solutions

Problem: 2-charge supertubes have 2 charges

$$S = 2\pi\sqrt{2N_1N_2} \qquad S_{TUBE} << S_{BH}$$

Marolf, Palmer; Rychkov

Solution:

- In deep scaling solutions: Bena, Bobev, Ruef, Warner

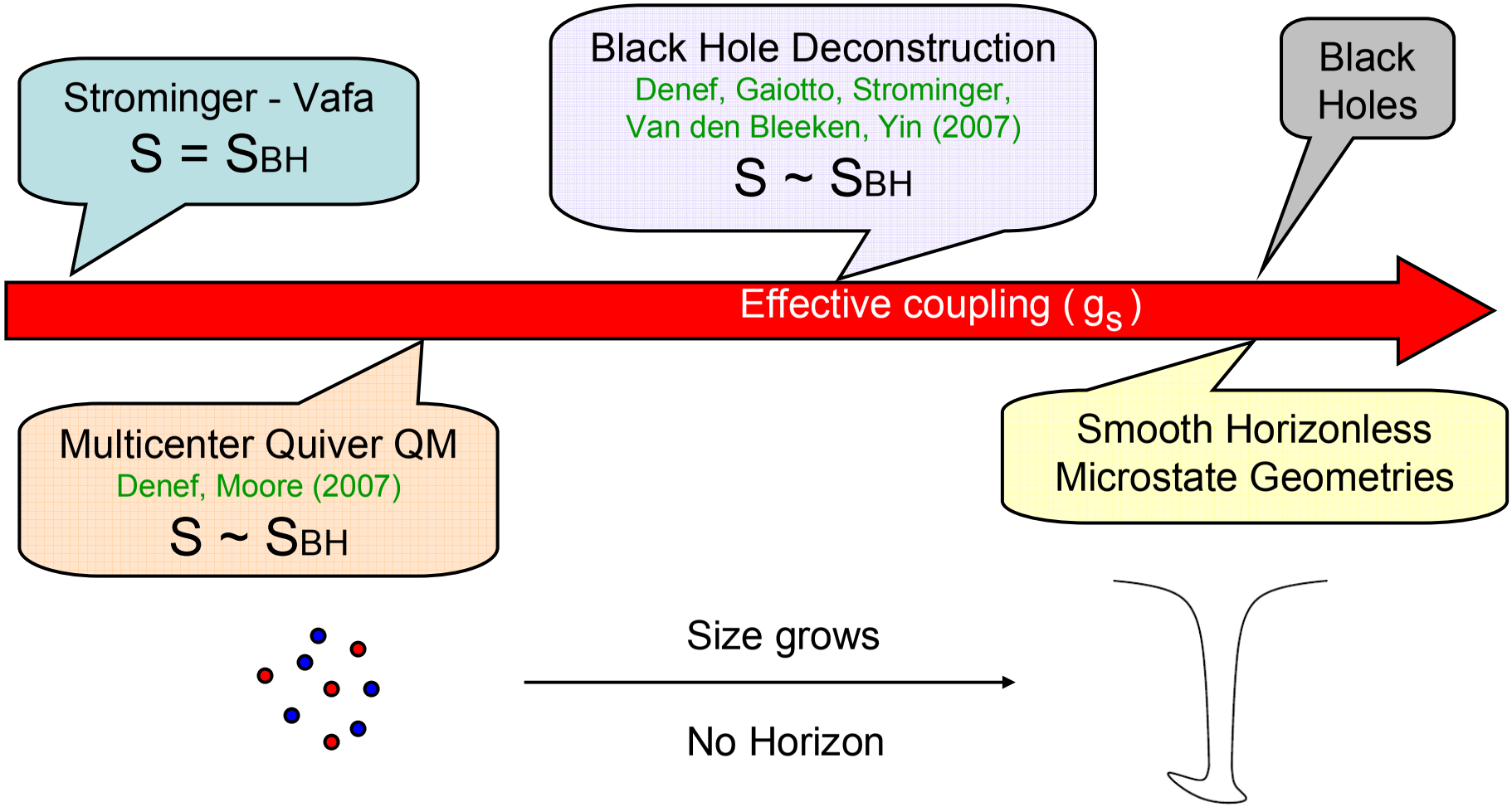
$$N_1 \to N_1^{\rm eff} \equiv N_1 + d_3 A_2, \quad N_2 \to N_2^{\rm eff} \equiv N_2 + d_3 A_1$$

- Entropy enhancement !!! FULL SOLUTION

$S_{TUBE}^{ENHANCED} \sim S_{BH}$ smooth sugra solutions + 1D

BPS microstates – the story:

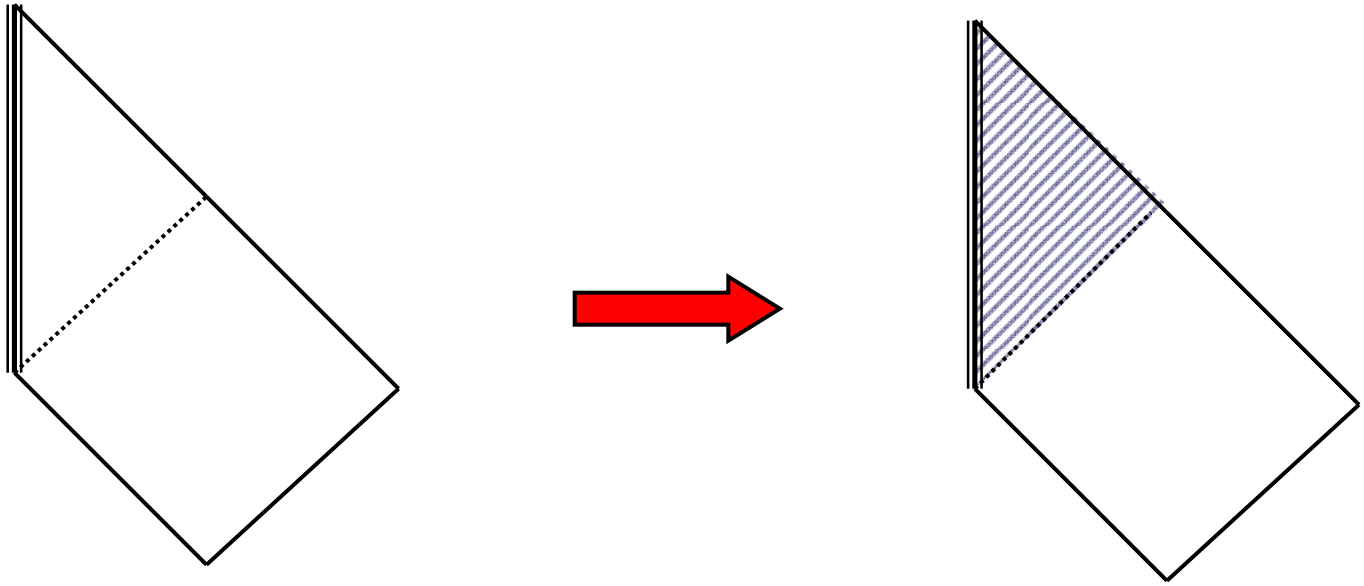
- We have a huge number of them
  - Arbitrary continuous functions
  - Infinite-dimensional moduli space
  - Supertube Entropy Enhancement
  - Black-Hole-like entropy ☺
    Bena, Bobev, Ruef and Warner
- Dual to CFT states ∈ typical sector
  - This is where BH states live too ☺
  - CFT perspective: highly weird if BH microstates were anything but fuzzballs
- Two non-backreacted calculations:
  - BH entropy from horizon-less scaling multicenter configurations ☺



Punchline: Typical states grow as G_N increases.
Horizon never forms.
Quantum effects from singularity extend to horizon

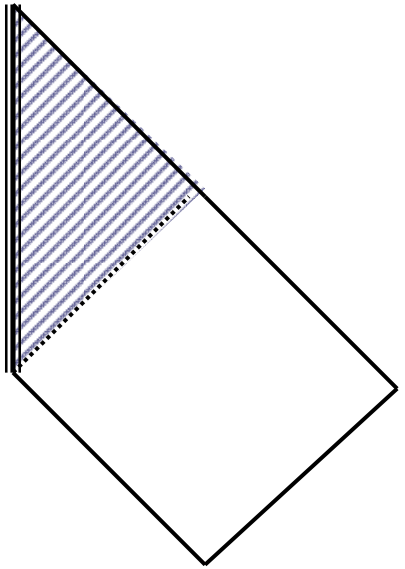
BPS Black Hole = Extremal

- This is not so strange
- Horizon in causal future of singularity
- Time-like singularity resolved by stringy low-mass modes extending to horizon



Always asked question:

- Why are quantum effects affecting the horizon (low curvature) ?
- Answer: space-time has singularity:
  - low-mass degrees of freedom
  - change physics on long distances
- Very common in string theory !!!
  - Polchinski-Strassler
  - Klebanov-Strassler
  - Giant Gravitons + LLM
  - D1-D5 system
- It can be even worse – quantum effects significant even without horizon or singularity !



de Boer, El Showk, Messamah, van den Bleeken

Extremal non-BPS black holes

- Same singularity. Same Penrose diagram.
  - Why BPS different from extremal ?
  - GR friends would strangle us.
- How can one show this ?
  - Construct multi-center extremal black holes
  - Easier said than done. No susy to help.
  - Second-order eqns. As messy as Kerr ☹
  - Could extremality make things easier ?
  - It does for single-center Ceresole, Dall'Agata + friends

Extremal non-BPS multicenter

Almost – BPS:

Goldstein, Katmadas:

$\mathbb{R}^4$ base (4D Hyper Kahler)

– $*G^I = G^I$

$d * dZ_1 = G^2 \wedge G^3$

– $d\vec{k} + *d\vec{k} = G^1 Z_1 + G^2 Z_2 + G^3 Z_3$

- Still solves equations of motion !!!
- Base needs just be Ricci-Flat
  Bena, Giusto, Ruef, Warner
- R^4 or R^3 x S^1 → BPS sols (absorb the –)
- Nontrivial new solutions in Taub-NUT

Why non-BPS ?

Bena, Dall'Agata, Giusto, Ruef, Warner

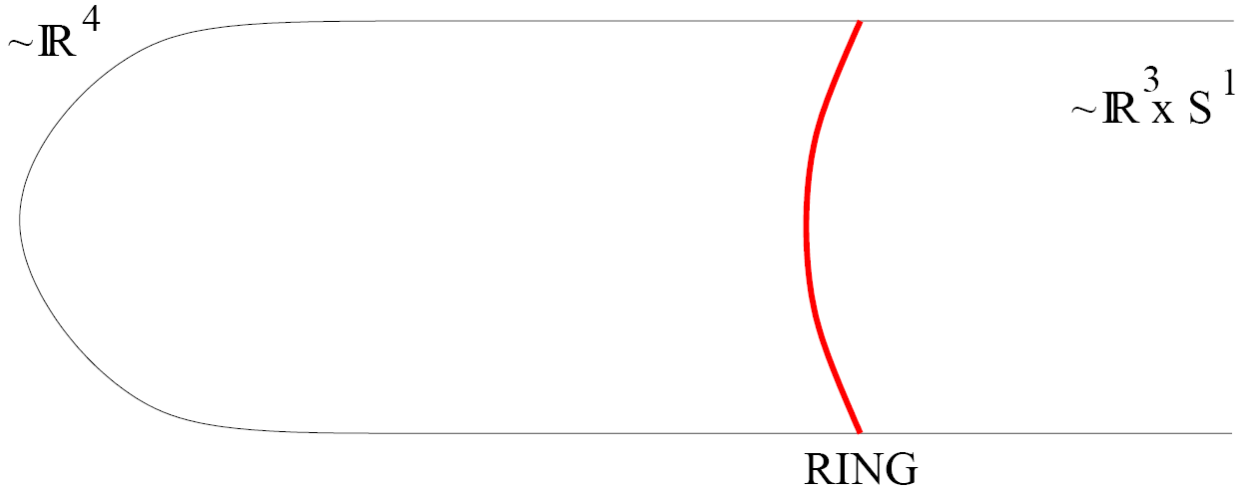
- Objects are locally BPS
  - M2 brane Killing Spinors
  - Incompatible with base space:

$$-*G^I = G^I$$

- Four-dimensional perspective
  - D2-D2-D2 system has 4 supercharges
  - Add D6 – susy preserved
  - Add anti-D6 – susy broken
  - Any 3 of the 4 branes preserve susy
- Probe D2 still feels no force
  - Built in the ansatz
  - Mutually BPS w.r.t. any of the background branes

Non-BPS Rings (in Taub-NUT)

Bena, Dall'Agata, Giusto, Ruef, Warner



$$S = \pi\sqrt{2n_1n_2\bar{N}_1\bar{N}_2 + 2n_1n_3\bar{N}_1\bar{N}_3 + 2n_2n_3\bar{N}_2\bar{N}_3 - n_1^2\bar{N}_1^2 - n_2^2\bar{N}_2^2 - n_3^2\bar{N}_3^2 - 4n_1n_2n_3J_T}$$

4D BH: D2 charges $\bar{N}_1\ \bar{N}_2\ \bar{N}_3$, D4 charges $n_1\ n_2\ n_3$ and D0 charge J_T

- Near-ring geometry same as BPS
- Distance from center of Taub-NUT different
- Almost-BPS equations more involved
- But we solved them …

A flavor of the equations

- Non-BPS

$$\Theta^{(I)} = d[K^{(I)}(d\psi + A) + b^{(I)}]$$

$$\Box_3 Z_I = V\frac{|\epsilon_{IJK}|}{2}\Box_3(K^{(J)}K^{(K)})$$

$$Z_I = L_I + \frac{|\epsilon_{IJK}|}{2}\sum_{j,k}\left(h + \frac{qr}{a_j a_k}\right)\frac{d_j^{(J)}d_k^{(K)}}{\Sigma_j\Sigma_k}$$

$$k = \mu(d\psi + A) + \omega$$

$$d(V\mu) + *_3 d\omega = V Z_I dK^{(I)}$$

- BPS

$$\Theta^{(I)} = d[\frac{K^{(I)}}{V}(d\psi + A) + b^{(I)}]$$

$$\Box_3 Z_I = V\frac{|\epsilon_{IJK}|}{2}\Box_3(\frac{K^{(J)}K^{(K)}}{V^2})$$

Simple solution:

$$Z_I = \frac{|\epsilon_{IJK}|}{2}\frac{K^{(J)}K^{(K)}}{V^2} + L_I$$

$$\mu = \frac{K^1K^2K^3}{V^2} + \frac{L_IK^I}{2} + M$$

How to solve for *k*

- Look at each term on the right hand side.
- Find corresponding μ and ω *e.g.*

$$d(V\mu_i^{(4)}) + *_3 d\omega_i^{(4)} = \frac{1}{r\Sigma_i} d\frac{1}{\Sigma_i} \qquad V\mu_i^{(4)} = \frac{\cos\theta}{2a_i\Sigma_i^2}, \quad \omega_i^{(4)} = \frac{r\sin^2\theta}{2a_i\Sigma_i^2} d\phi$$

- Combine all pieces to find full *k*

$$\mu = \sum_i \ell_I d_i^{(I)} \mu_i^{(1)} + \sum_i Q_i^{(I)} d_i^{(I)} (h\mu_i^{(2)} + q\mu_i^{(4)}) + \sum_{i\neq i'} Q_i^{(I)} d_{i'}^{(I)} (h\mu_{ii'}^{(3)} + q\mu_{ii'}^{(5)})$$
$$+ \sum_{i,j,k} d_i^{(1)} d_j^{(2)} d_k^{(3)} (h^2\mu_{ijk}^{(6)} + q^2\mu_{ijk}^{(7)} + hq\mu_{ijk}^{(8)}) + \mu^{(9)}$$

$$\omega = \sum_i \ell_I d_i^{(I)} \omega_i^{(1)} + \sum_i Q_i^{(I)} d_i^{(I)} (h\omega_i^{(2)} + q\omega_i^{(4)}) + \sum_{i\neq i'} Q_i^{(I)} d_{i'}^{(I)} (h\omega_{ii'}^{(3)} + q\omega_{ii'}^{(5)})$$
$$+ \sum_{i,j,k} d_i^{(1)} d_j^{(2)} d_k^{(3)} (h^2\omega_{ijk}^{(6)} + q^2\omega_{ijk}^{(7)} + hq\omega_{ijk}^{(8)}) + \omega^{(9)},$$

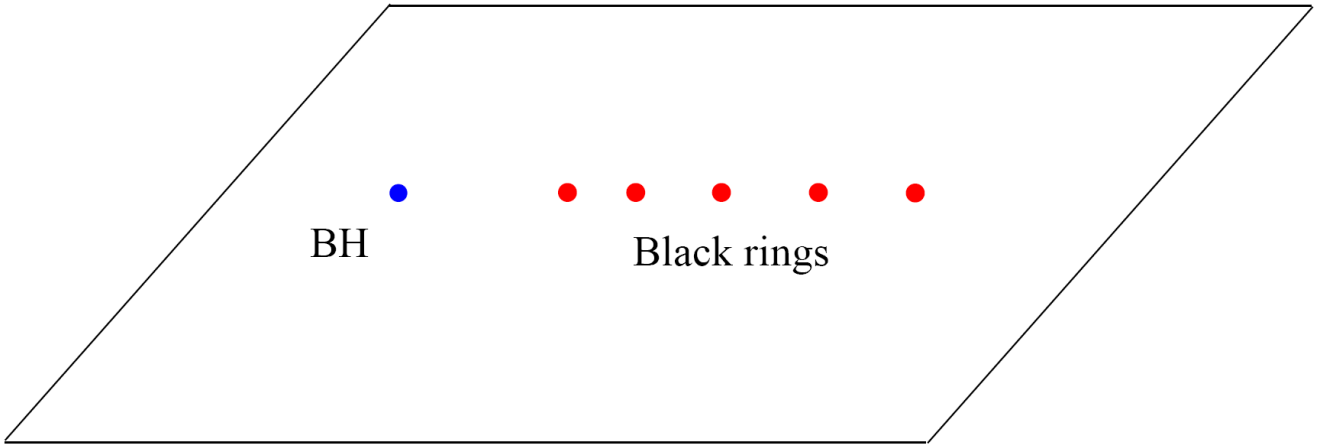
Regularity

- Vanishing ω on axis $\rightarrow$ no Dirac strings
  - Implies "bubble" or "integrability equations"
  - Walls of marginal stability, wall crossing, etc
  - Bubble equations now cubic !!!
- No CTC's at black ring horizons $\rightarrow$ free harmonic function in *k* has dipole part
- Add dipole part at Taub-NUT center $\rightarrow$ most general under-rotating non-BPS extremal 4D BH Bena, Dall'Agata, Giusto, Ruef, Warner

Multiple non-BPS Rings

Bena, Giusto, Ruef, Warner

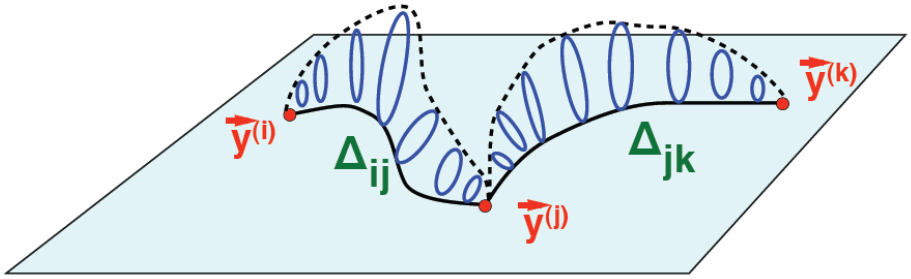
- 4D multicentered black holes
- BH at tip of Taub-NUT: anti-D6 + D2 + rotation
- BH's coming from rings: D4-D2-D0 charges



- There exist scaling solutions (very long throats)
- Scaling solutions can have nonzero angular momentum !
- In scaling limit: non-BPS bubble eqs = BPS bubble eqs

Non-BPS horizonless microstates

- Impossible within almost-BPS ansatz
- Anti-self-dual flux in GH spaces is non-normalizable
- Solution not asymptotically-flat
- We have to be smarter !
- 2 ways



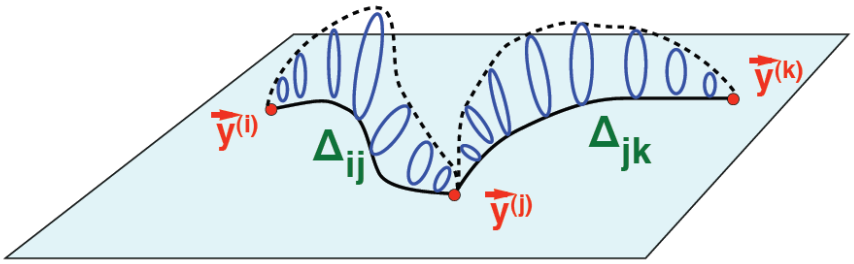
1. Floating Branes

Need good grad student + postdocs

- Blast full force through sugra equations
  - Particular ansatz – M2 branes feel no force
  - Warp factors equal to electric potentials
- New class of 5D solutions: **Bena, Giusto, Ruef, Warner**
- 4D base = electrovac Euclidean solution
- Any electrovac sol. → full solution
- Linear procedure !
- Two obvious families of base-spaces
  - Israel-Wilson metrics
  - Euclidean Kerr-Newman **Bobev, Ruef**

1. Floating Branes

- Israel-Wilson base



$$ds_4^2 = (V_+ V_-)^{-1}(d\psi + \vec{A}\cdot d\vec{y})^2 + (V_+ V_-)(dy_1^2 + dy_2^2 + dy_3^2)$$

$$\vec{\nabla}\times\vec{A} = V_-\vec{\nabla}V_+ - V_+\vec{\nabla}V_-$$

- Highly nontrivial non-BPS solutions:
  - D6-D4-D2-D0 BH in anti-D6 background
  - Can put normalizable flux on cycle between anti-D6 and D6

2. Spectral Flow

- Sequence of dualities: Bena, Bobev, Warner
  - Lift to D1-D5-P duality frame (6D sugra)
  - Mix KK U(1) with GH U(1) + dualize back
- Reshuffles charges. BPS $\Leftrightarrow$ BPS
- Almost-BPS sols $\Leftrightarrow$ IW base solutions

  Bena, Giusto, Ruef, Warner
- GH center (D6) $\Leftrightarrow$ 2-charge supertube (D4)
- Almost-BPS supertube $\Leftrightarrow$ IW solution with flux between D6 and anti-D6
- Several supertubes $\Leftrightarrow$ smooth multicenter deep scaling solution (as good as it gets)

Physics punchline

In string theory:

extremal black holes = ensembles of horizonless microstates.

- Can be proven (disproven) rigorously
- No spacetime inside horizon. Instead – quantum superposition of microstates
- No unitarity loss/information paradox

What about nonextremal ?

- Given total energy budget:

  most entropy obtained by making
  brane-antibrane pairs
- $S=2\pi\ (N_1^{1/2}+\underline{N}_1^{1/2})(N_2^{1/2}+\underline{N}_2^{1/2})(N_3^{1/2}+\underline{N}_3^{1/2})$

  Horowitz, Maldacena, Strominger
- Mass gap = $1/N_1N_2N_3$
- Extend on long distances (horizon scale)
- More mass – lower mass-gap – larger size
- Only 3 solutions known. Work very nicely.

  JejjalaMadden Ross Titchener (JMaRT); Myers & al, Mathur & al.
- One multi-center solution (with horizon)

  Camps, Emparan, Figueras, Giusto, Saxena

The Running Bolt

- Base space just needs be Ricci Flat
- Take Euclidean Schwarzschild:

$$ds_4^2 = \left(1-\frac{2m}{r}\right)d\tau^2 + \left(1-\frac{2m}{r}\right)^{-1}dr^2 + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2$$

- Put fluxes on the Bolt — Bena, Giusto, Ruef, Warner
  - Bolt starts running ☺
- Smooth solution.
- D4, D2, D0, mass of big fat nonextremal BH !

$$M = \frac{\pi}{4G_5}\left(16m^2 + \frac{\varepsilon}{4\pi^2}(Q^1+Q^2+Q^3)\right)$$

- Mass decreases with increasing $|\mathcal{Q}|$!

Summary and Future Directions

- Strong evidence that in string theory: extremal black holes = ensembles of microstates
  - QG low-mass modes can change physics on large (horizon) scales
  - Convergence of research directions
- Classify all extremal solutions
  - 2, 3 spectral flows take us out of IW ansatz
  - Solve equations. Find larger (largest ?) class.
  - What is the orbit of multi-center extremal BH's ?
- Extend to non-extremal black holes
  - Probably; at least near-extremal
  - Need more non-extremal microstates
    - Inverse scattering methods. Use JMaRT as seed.
- New light degrees of freedom at horizon. Experiment ?